

# D.C. Motors

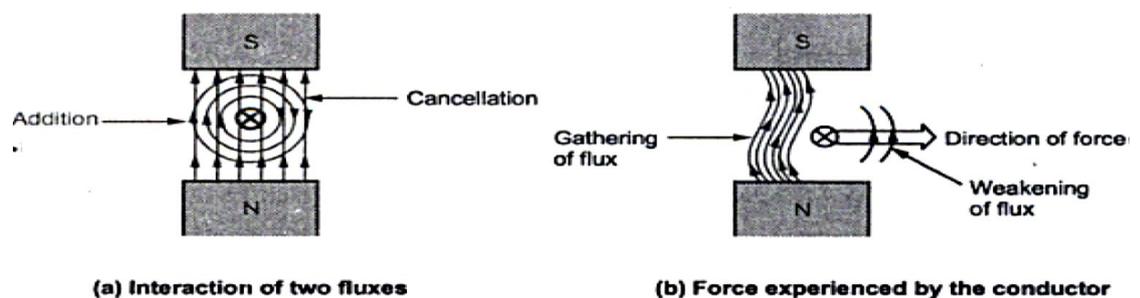
## Introduction

A motor is a device which converts an electrical energy into the mechanical energy. The energy conversion process is exactly opposite to that involved in a d.c. generator. In a generator the input mechanical energy is supplied by a prime mover while in a d.c. motor, input electrical energy is supplied by a d.c. supply. The construction of a d.c. machine is same whether it is a motor or a generator.

## 1. Principle of Operation of a D.C. Motor

The principle of operation of a d.c. motor can be stated in a single statement as 'when a current carrying conductor is placed in a magnetic field; it experiences a mechanical force'. In a practical d.c. motor, field winding produces a required magnetic field while armature conductors play a role of a current carrying conductors and hence armature conductors experience a force. As conductors are placed in the slots which are on the periphery, the individual force experienced by the conductors acts as a twisting or turning force on the armature which is called a *torque*. The torque is the product of force and the radius at which this force acts. So overall armature experiences a torque and starts rotating.

Consider a single conductor placed in a magnetic field, it is clear that on one side of the conductor, both the fluxes are in the same direction. In this case, on the left of the conductor there is gathering of the flux lines as two fluxes help each other. As against this, on the right of the conductor, the two fluxes are in opposite direction and hence try to cancel each other. Due to this, the density of the flux lines in this area gets weakened. So on the left, there exists high flux density area while on the right of the conductor there exists low flux density area as shown in the **Figure (1)**.



**Figure (1)**

This flux distribution around the conductor acts like a stretched rubber band under tension. This exerts a mechanical force on the conductor which acts from high flux density area towards low flux density area, i.e. from left to right for the case considered as shown in the **Figure (1-b)**.

**Note:** It should be noted that the function of a commutator in the motor is the same as in a generator. By reversing current in each conductor as it passes from one pole to another, it helps to develop a continuous and unidirectional torque.



## 2. Direction of Rotation of Motor

The magnitude of the force experienced by the conductor in a motor is given by,

$$F = B l I \quad \text{Newtons (N)}$$

$B$  = Flux density due to the flux produced by the field winding.

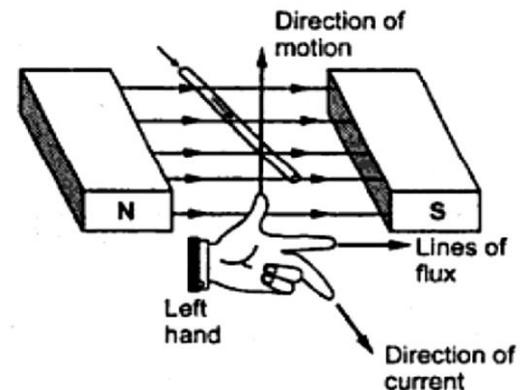
$l$  = Active length of the conductor.

$I$  = Magnitude of the current passing through the conductor.

The direction of such force i.e. the direction of rotation of a motor can be determined by *Fleming's left hand rule*. So *Fleming's right hand rule* is to determine direction of induced e.m.f. i.e. for generating action while *Fleming's left hand rule* is to determine direction of force experienced i.e. for motoring action.

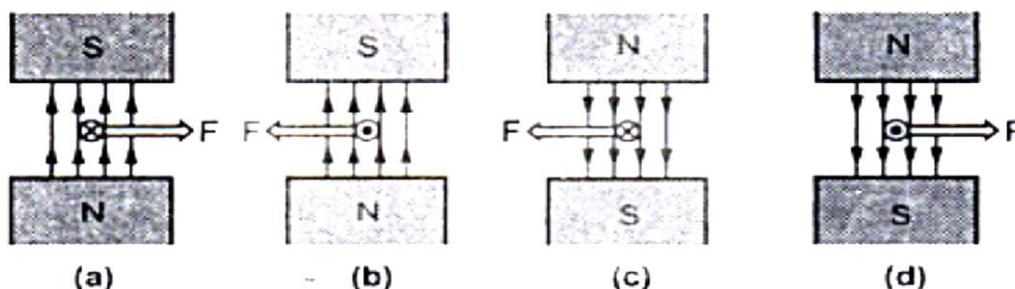
**Fleming's Left Hand Rule** states that, 'Outstretch the three fingers of the left hand namely the first finger, middle finger and thumb such that they are mutually perpendicular to each other. Now point the first finger in the direction of magnetic field and the middle finger in the direction of the current then the thumb gives the direction of the force experienced by the conductor.'

The Fleming's left hand rule can be diagrammatically shown as in the **Figure (2)**.



**Figure (2):** Fleming's left hand rule

It can be seen from the **Figure (3)** that if the direction of the main field in which current carrying conductor is placed, is reversed, force experienced by the conductor reverses its direction. Similarly keeping main flux direction unchanged, the direction of current passing through the conductor is reversed, the force experienced by the conductor reverses its direction. However if both the directions are reversed, the direction of the force experienced remains the same.



**Figure (3):** Direction of force experienced by conductor



**Note :** So in a practical motor, to reverse its direction of rotation, either direction of main field produced by the field winding is reversed or direction of the current passing through the armature is reversed.

The direction of the main field can be reversed by changing the direction of current passing through the field winding, which is possible by interchanging the polarities of supply which is given to the field winding.

### 3. Significance of Back E.M.F.

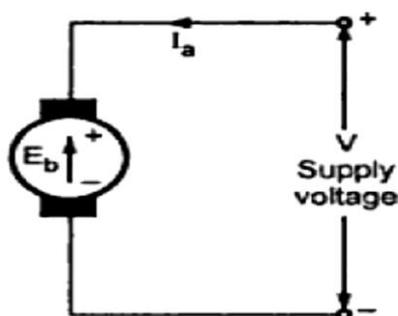
It is seen in the generating action, that when a conductor cuts the lines of flux, e.m.f. gets induced in the conductor.

\*The question is obvious that in a d.c. motor, after a motoring action, armature starts rotating and armature conductors cut the main flux. So is there a generating action existing in a motor ?

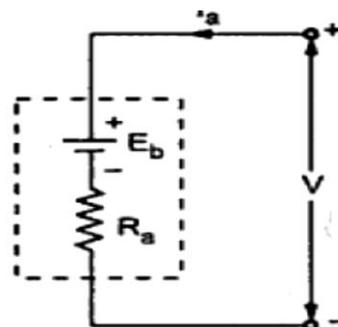
\*The answer to this question is Yes.

After a motoring action, there exists a generating action. There is an induced e.m.f. in the rotating armature conductors according to *Faraday's law of electromagnetic induction*. This induced e.m.f. in the armature always acts in the opposite direction of the supply voltage. This is according to the *Lenz's law* which states that the direction of the induced e.m.f. is always so as to oppose the cause producing it. In a d.c. motor, electrical input i.e. the supply voltage is the cause and hence this induced e.m.f. opposes the supply voltage. This e.m.f. tries to set up a current through the armature which is in the opposite direction to that, which supply voltage is forcing through the conductor. So as this e.m.f. always opposes the supply voltage, it is called *back e.m.f.* and denoted as  $E_b$ , its magnitude can be determined by the e.m.f. equation. So,

$$E_b = \frac{\Phi Z N P}{60 A} \text{ volt}$$



(a) Back e.m.f. in a d.c motor



(b) Equivalent circuit

Figure (4)



This e.m.f. is shown schematically in the Figure (4-a). So if  $V$  is supply voltage in volts and  $R_a$  is the value of the armature resistance, the equivalent electric circuit can be shown as in the Figure (4-b).

**Note:** the back e.m.f. is always less than supply voltage ( $E_b < V$ ).

#### 4. Types of D.C. Motors

Similar to the d.c. generators, the d.c. motors are classified depending upon the way of connecting the field winding with the armature winding. The different types of d.c. motors are:

- (a) Permanent magnet d.c. motor.
- (b) Electromagnet d.c. motor
  - (1) Separately-excited d.c. motor.
  - (2) Self-excited d.c. motor ( shunt, series and compound motors).

##### (a) Permanent-Magnet D.C Motor

The permanent-magnet D.C motor shown in Figure (5), is construct in the same manner as D.C generator.

When this type of motor is used, the D.C power supply is connected directly to the armature conductors through the brush to commutator assembly. The magnetic field is produced by permanent magnets mounted on the stator (yoke).

The permanent-magnet motor has several advantages over conventional types of D.C motors. The advantage is a reduced operational **cost**, and The **direction** of **rotation** of a permanent magnet motor can be **reversed** by reversing the source polarities. The speed of the permanent-magnet motor are similar to those of the shunt wound D.C motor.

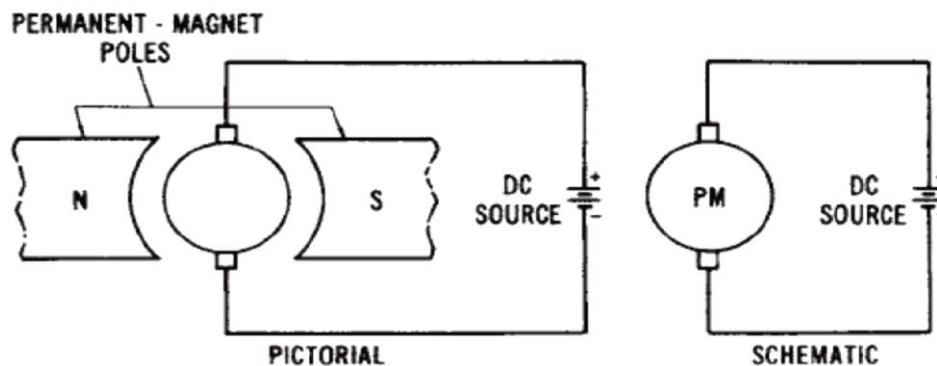


Figure (5)



## (2) Self Excited d.c. Motor-Magnet D.C Motor

### (i) D.C. Shunt Motor

The field winding is connected across the armature winding and the combination is connected across the supply, as shown in the Figure (6). The shunt field winding has more number of turns with less cross-sectional area.

$$I_L = I_a + I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}}$$

$$V = E_b + I_a R_a + V_{brush}$$

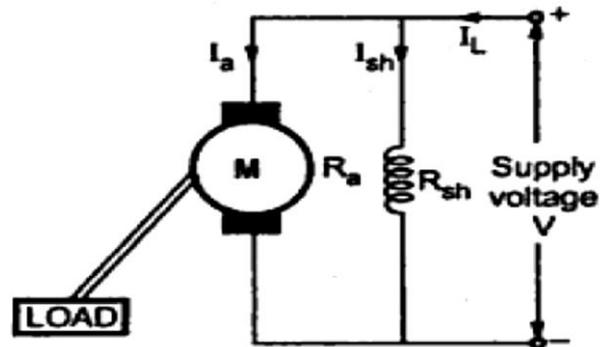


Figure (6): Shunt motor

**Note:** As long as supply voltage is constant, the flux produced is constant. Hence dc shunt motor is called constant flux motor. So flux produced by the field winding is proportional to the current passing through it i.e.  $I_{sh}$ .

$$\Phi \propto I_{sh}$$

### (ii) D.C. Series Motor

In this type of motor, the series field winding is connected in series with the armature and the supply, as shown in the Figure (7). The value of series field winding resistance is very small and it is made of small number of turns having large cross-sectional area.

$$I_L = I_a = I_{se}$$

$$V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$$

$$V = E_b + I_a (R_a + R_{se}) + V_{brush}$$

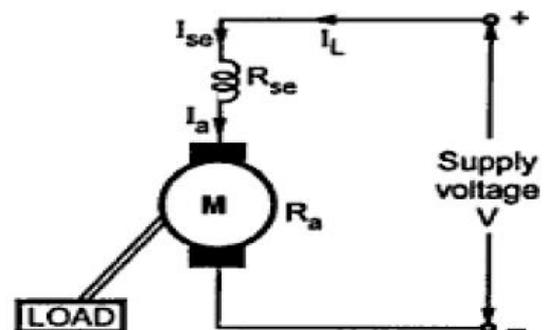


Figure (7): Series motor

**Note:** In series motor, entire armature current is passing through the series field winding. So flux produced is proportional to the armature current

$$\Phi \propto I_{se} \propto I_a \quad (\text{for series motor})$$

### (iii) DC Compound Motor

The compound motor consists of series and shunt field windings. It is classified as long and short shunt compound motor.

#### (a) Long Shunt Compound Motor

In this type, the shunt field winding is connected across the combination of armature and the series field winding as shown in Figure (8).

$$I_L = I_{sh} + I_{se}$$

$$I_a = I_{se}$$

$$I_{sh} = \frac{V}{R_{sh}}$$

$$V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$$

$$V = E_b + I_a (R_a + R_{se}) + V_{brush}$$

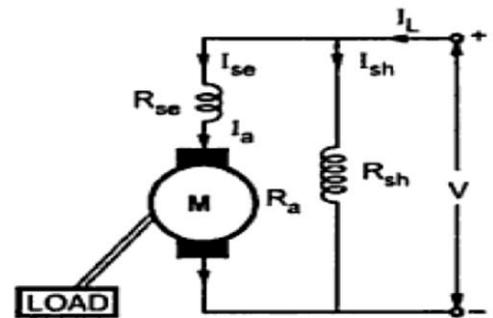


Figure (8): Long shunt compound motor

#### (b) Short Shunt Compound Motor

In this type, shunt field winding is connected in parallel with armature winding & series field is connected in series with this combination, as shown in Figure (9).

$$I_L = I_{se}$$

$$I_L = I_a + I_{sh}$$

$$I_{sh} = \frac{V - I_L R_{se}}{R_{sh}}$$

$$V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$$

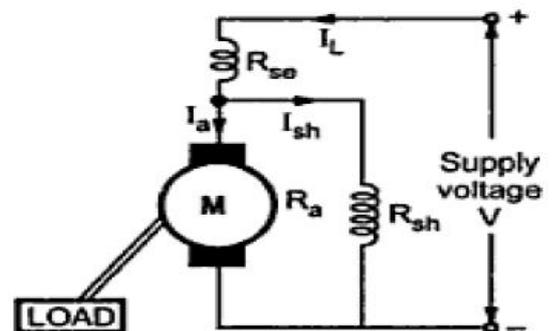


Figure (9): Short shunt compound motor

**Note:** If the two field windings are wound in such a manner that the fluxes produced by the two always help each other, the motor is called *cumulatively compound*. If the fluxes produced by the two field windings are trying to cancel each other (they are in opposite direction), the motor is called *differential compound*.

A long shunt compound motor can be of cumulative or differential type. Similarly short shunt compound motor can be cumulative or differential type.



### 5. Power Equation of a D.C. Shunt Motor

The voltage equation of a d.c motor is given by,

$$V = E_b + I_a R_a$$

Multiplying both sides of the above equation by  $I_a$  we get

$$V I_a = E_b I_a + (I_a)^2 R_a$$

This equation is called power equation of a d.c. motor.

$V I_a$  = Net electrical power input to the armature measured in watts.

$(I_a)^2 R_a$  = Power loss due to the resistance of the armature called armature copper loss.

$E_b I_a$  = is called electrical equivalent of gross mechanical power ( $P_m$ ) developed by the armature.

$\therefore$  Power input to the armature - Armature copper loss = Gross mechanical power developed in the armature.

### 6. Condition for Maximum Power for Shunt Motor

For a motor from power equation it is known that,

$$P_m = \text{Gross mechanical power developed} = E_b I_a = V I_a - (I_a)^2 R_a$$

For maximum  $P_m$ ,  $\frac{dP_m}{dI_a} = 0$

$$\therefore 0 = V - 2I_a R_a$$

$$I_a = \frac{V}{2R_a} \quad \text{i.e.} \quad I_a R_a = \frac{V}{2}$$

Substituting in voltage equation,

$$V = E_b + I_a R_a = V = E_b + (V/2)$$

$$E_b = \frac{V}{2} \quad \dots \text{Condition for maximum power}$$

**Note:** This is practically impossible to achieve as for this  $E_b$ , current required is much more than its normal rated value, large heat will be produced and efficiency of motor will be less than 50 %.



**Example 1:** A 220-V d.c. shunt machine has an armature resistance of  $0.5 \Omega$ . If the full-load armature current is 20 A, find the induced e.m.f. when the machine acts as (i) generator (ii) motor.

**Solution:**

In each case, shunt current is considered negligible because its value is not given.

(a) As Generator [Figure (10-a)]  $E_g = V + I_a R_a = 220 + 0.5 \times 20 = 230 \text{ V}$

(b) As Motor [Figure (10-b)]  $E_b = V - I_a R_a = 220 - 0.5 \times 20 = 210 \text{ V}$

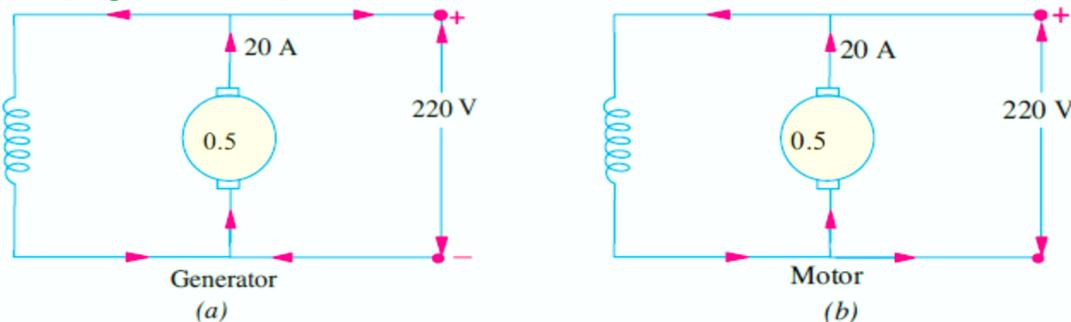


Figure (10)

**Example 2:** A 440-V, shunt motor has armature resistance of  $0.8 \Omega$  and field resistance of  $200 \Omega$ . Determine the back e.m.f. when giving an output of 7.46 kW at 85 percent efficiency.

**Solution:**

$$\% \eta = \frac{P_{\text{output}}}{P_{\text{input}}} \implies \therefore \text{Motor input power } (P_{\text{input}}) = (7.46 \times 10^3) / 0.85 = 8776.5 \text{ W}$$

$$P_{\text{input}} = I_L V \implies \text{Motor input current } (I_L) = \frac{8776.5}{440} = 19.95 \text{ A}$$

$$I_{\text{sh}} = 440 / 200 = 2.2 \text{ A}$$

$$I_a = I_L - I_{\text{sh}} = 19.95 - 2.2 = 17.75 \text{ A}$$

$$\text{Now, } E_b = V - I_a R_a = 440 - (17.75 \times 0.8) = 425.8 \text{ V}$$

**Example 3:** A 25-kW, 250-V, d.c. shunt generator has armature and field resistances of  $0.06 \Omega$  and  $100 \Omega$  respectively. Determine the total armature power developed when working (i) as a generator delivering 25 kW output and (ii) as a motor taking 25 kW input.

**Solution:**

**As Generator** [Figure (11-a)]

$$P_{\text{out}} = 25 \text{ kW}$$

$$P_{\text{out}} = I_L V$$

$$\therefore \text{Output current } (I_L) = 25,000 / 250 = 100 \text{ A}; I_{\text{sh}} = 250 / 100 = 2.5 \text{ A}$$

$$I_a = I_L + I_{\text{sh}} = 102.5 \text{ A}$$

$$E_g = V + I_a R_a = 250 + (102.5 \times 0.06) = 256.15 \text{ V}$$

$$\text{Power developed in armature} = E_g I_a = 256.15 \times 102.5$$

$$= 26250 \text{ W} = 26.25 \text{ kW}$$

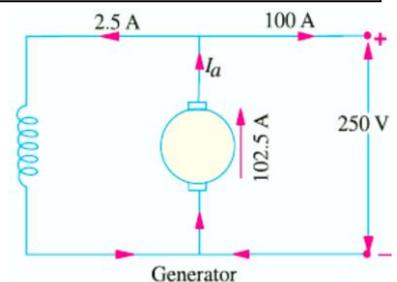


Figure (11-a)



**As Motor** [Figure (11-b)]

$$P_{\text{input}} = 25 \text{ kW}$$

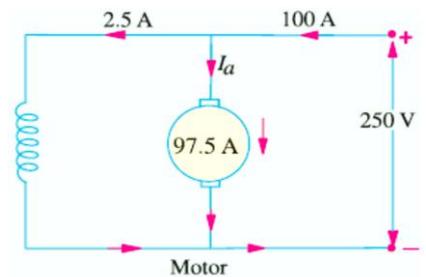
$$P_{\text{input}} = I_L V$$

$$\text{Motor input current } (I_L) = 100 \text{ A} ; I_{\text{sh}} = 2.5 \text{ A}$$

$$I_a = I_L - I_{\text{sh}} = 97.5 \text{ A}$$

$$E_b = V - I_a R_a = 250 - (97.5 \times 0.06) = 250 - 5.85 = 244.15 \text{ V}$$

$$\begin{aligned} \text{Power developed in armature} &= E_b I_a = 244.15 \times 97.5 \\ &= \mathbf{23800 \text{ W} = 23.8 \text{ kW}} \end{aligned}$$



**Figure (11-b)**

**Example 4:** A 4 pole, 32 conductor, lap-wound d.c. shunt generator with terminal voltage of 200 volts delivering 12 amps to the load has  $R_a = 2$  and field circuit resistance of 200 ohms. It is driven at 1000 r.p.m. Calculate the flux per pole in the machine. If the machine has to be run as a motor with the same terminal voltage and drawing 5 amps from the mains, maintaining the same magnetic field, find the speed of the machine.

**Solution:**

**As Generator** [Figure (12-a)]

$$I_{\text{sh}} = \frac{V}{R_{\text{sh}}} = \frac{200}{200} = 1 \text{ A}, I_L = 12 \text{ A}$$

$$I_a = I_L + I_{\text{sh}} = 12 + 1 = 13 \text{ A}$$

$$E_g = V + I_a R_a = 200 + (13 \times 2) = 226 \text{ V}$$

$$E_g = \frac{\Phi Z N P}{60 A} \implies 226 = \frac{\Phi \times 32 \times 1000}{60} \frac{4}{4} \implies \therefore \Phi = 0.42375 \text{ wb}$$

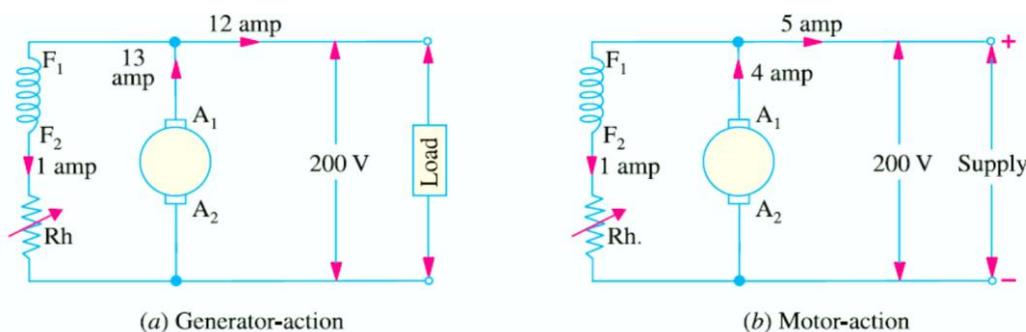
**As Motor** [Figure (12-b)]

$$I_{\text{sh}} = \frac{V}{R_{\text{sh}}} = \frac{200}{200} = 1 \text{ A}, I_L = 5 \text{ A}$$

$$I_a = I_L - I_{\text{sh}} = 5 - 1 = 4 \text{ A}$$

$$E_b = V - I_a R_a = 200 - (4 \times 2) = 192 \text{ V}$$

$$E_b = \frac{\Phi Z N P}{60 A} \implies 192 = \frac{0.42375 \times 32 \times N}{60} \frac{4}{4} \implies \therefore N = 850 \text{ r.p.m.}$$



**Figure (12)**



## Tutorial Problems (1)

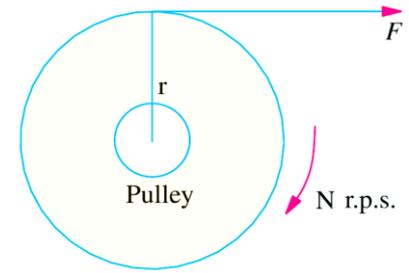
- [1] What do you understand by the term 'back e.m.f.' ? A d.c. motor connected to a 460-V supply has an armature resistance of  $0.15 \Omega$ . Calculate
- (a) The value of back e.m.f. when the armature current is 120 A.  
(b) The value of armature current when the back e.m.f. is 447.4 V.    [(a) 442 V (b) 84 A]
- [2] A d.c. motor connected to a 460-V supply takes an armature current of 120 A on full load. If the armature circuit has a resistance of  $0.25 \Omega$ , calculate the value of the back e.m.f. at this load.    [430 V]
- [3] A 4-pole d.c. motor takes an armature current of 150 A at 440 V. If its armature circuit has a resistance of  $0.15 \Omega$ , what will be the value of back e.m.f. at this load ?    [417.5 V]



### 7. Torque Equation of a D.C. Motor

Torque ( $T$ ) is meant the turning or twisting moment of a force about an axis. It is measured by the product of the force and the radius at which this force acts.

Consider a pulley of radius  $r$  metre acted upon by a circumferential force of  $F$  Newton which causes it to rotate at  $N$  in r.p.m. **Figure (13)**.



**Figure (13)**

$$T = F \times r \quad \text{Newton-metre (N - m)}$$

Work done by this force in one revolution = Force  $\times$  distance =  $F \times 2\pi r$  Joule

$$\begin{aligned} \text{Power developed} &= F \times 2\pi r \times N \quad \frac{\text{Joule}}{\text{Second}} \quad \text{or Watt} \\ &= (F \times r) \times 2\pi N \quad \text{Watt} \\ &= T \times \omega \quad \text{Watt} \end{aligned}$$

$$\therefore \quad P = T \omega \quad \text{Watt}$$

Where  $\omega = 2\pi N$ , ( $\omega$ ) is angular velocity in radian/second

Moreover, if  $N$  is in r.p.m., then  $\omega = 2\pi N/60$  rad/s

$$\therefore \quad P = \frac{2\pi N}{60} \times T = \frac{NT}{9.55}$$

### 8. Armature Torque ( $T_a$ ) of a Motor

If ( $T_a$ ) is in N-m of a motor running at ( $N$ ) r.p.s. then,

$$\text{power developed} = T_a \omega = T_a \times (2\pi N) \text{ watt} \quad \dots(i)$$

We also know that electrical power converted into mechanical power in the armature is,

$$= E_b I_a \text{ watt} \quad \dots(ii)$$

$$\text{Equating (i) and (ii), we get } T_a \times (2\pi N) = E_b I_a \quad \dots(iii)$$

Since  $E_b = \Phi Z N \frac{P}{A}$  volt, we have

$$T_a \times 2\pi N = \Phi Z N \frac{P}{A} I_a$$

$$\text{or } T_a = \frac{1}{2\pi} \Phi Z I_a \left( \frac{P}{A} \right) \text{ N-m}$$

$$\therefore \quad T_a = 0.159 \Phi Z I_a \times (P/A) \quad \text{N-m}$$



**Note.** From the above equation for the torque, we find that  $T_a \propto \Phi I_a$ .

(a) In the case of a series motor,  $\Phi$  is directly proportional to  $I_a$  (before saturation) because field windings carry full armature current  $\therefore T_a \propto (I_a)^2$

(b) For shunt motors,  $\Phi$  is practically constant, hence  $T_a \propto I_a$ .

As seen from (iii) above, 
$$T_a = \frac{E_b I_a}{\frac{2\pi N}{60}} \text{ N-m (N in r.p.s.)}$$

If N is in r.p.m., then, 
$$T_a = \frac{E_b I_a}{\frac{2\pi N}{60}} = \frac{60}{2\pi} \frac{E_b I_a}{N} = 9.55 \frac{E_b I_a}{N} = 9.55 \frac{P_m}{N} \text{ N-m}$$

### 9. Shaft Torque ( $T_{sh}$ )

The whole of the armature torque, as calculated above, is not available for doing useful work, because a certain percentage of it is required for supplying iron and friction losses in the motor.

The torque which is available for doing useful work is known as shaft torque  $T_{sh}$ . It is so called because it is available at the shaft. The motor output is given by,

Output power ( $P_{out}$ ) =  $T_{sh} \times 2\pi N$  Watt where ( $T_{sh}$  is in N-m and N in r.p.s.)

$$\begin{aligned} \therefore T_{sh} &= \frac{P_{out}}{\omega} = \frac{P_{out}}{2\pi N} \text{ N-m} && \text{where } (P_{out} \text{ in watt and } N \text{ in r.p.s.)} \\ &= \frac{P_{out}}{2\pi \left(\frac{N}{60}\right)} \text{ N-m} && \text{where } (P_{out} \text{ in watt and } N \text{ in r.p.m.)} \\ &= \frac{60}{2\pi} \frac{P_{out}}{N} = 9.55 \times \frac{P_{out}}{N} \text{ N-m} \end{aligned}$$

The difference ( $T_a - T_{sh}$ ) is known as lost torque ( $T_f$ ) and is due to iron and friction losses of the motor.

$$T_a = T_f + T_{sh}$$

$$\text{Net output of motor} = P_{out} = T_{sh} \times \omega$$

**Note:**

$$T_a = \frac{P_m}{\omega}, T_{sh} = \frac{P_{out}}{\omega}, T_f = \frac{W_{stray}}{\omega}, \text{ where } (\omega = 2\pi N) \text{ \& } (N) \text{ in r.p.s.}$$



### 10. No Load Condition of a Shunt Motor

On no load, the load requirement is absent, so  $T_{sh} = 0$ .

So on no load, motor keeps on rotating at a speed of  $N_0$  r.p.m. drawing an armature current of  $I_{a0}$ . This is just enough to produce a torque  $T_{a0}$  which satisfies the friction, windage and iron losses of the motor. On no load, speed of the motor is large hence  $(E_b)$  is also large hence  $(V - E_{b0})$  is very small hence armature current  $I_{a0}$  is also small. So motor draws less current on no load and takes more and more current as motor load increases.

### 11. Torque and Speed Equations

Before analyzing the various characteristics of motors, let us revise the torque and speed equations as applied to various types of motors.

$$T \propto \Phi I_a \quad \text{from torque equation.}$$

This is because  $0.159 \frac{PZ}{A}$  is a constant for a given motor.

Now  $\Phi$  is the flux produced by the field winding and is proportional to the current passing through the field winding.

$$\Phi \propto I_{\text{field}}$$

But for various types of motors, current through the field winding is different. Accordingly torque equation must be modified.

For a d.c shunt motor,  $I_{sh}$  is constant as long as supply voltage is constant. Hence  $\Phi$  flux is also constant.

$$T \propto I_a \quad \text{for shunt motor.}$$

For a d.c. series motor,  $I_{se}$  is same as  $I_a$ . Hence flux  $\Phi$  is proportional to the armature current  $I_a$ .

$$T \propto I_a \Phi \propto (I_a)^2 \quad \text{for series motor.}$$

Similarly as  $E_b = \frac{\Phi P N Z}{60 A}$ , we can write the speed equation as,

$$E_b \propto \Phi N$$

$$N \propto \frac{E_b}{\Phi}$$



But  $V = E_b + I_a R_a$  neglecting brush drop

$$E_b = V - I_a R_a$$

∴ Speed equation becomes,

$$N \propto \frac{V - I_a R_a}{\phi}$$

So for shunt motor as flux  $\phi$  is constant,

$$\therefore N \propto V - I_a R_a$$

While for series motor, flux  $\phi$  is proportional to  $I_a$ ,

$$\therefore N \propto \frac{V - I_a R_a - I_a R_{se}}{I_a}$$

These relations play an important role in understanding the various characteristics of different types of motors.

## 12. Speed Regulation

The speed regulation for a d.c motor is defined as the ratio of change in speed corresponding to no load and full load condition to speed corresponding to full load.

Mathematically it is expressed as.

$$\% \text{ speed regulation} = \frac{N_{no \text{ load}} - N_{full \text{ load}}}{N_{full \text{ load}}} \times 100$$

**Example 5:** A d.c. motor takes an armature current of 110 A at 480 V. The armature circuit resistance is 0.2 Ω. The machine has 6-poles and the armature is lap-connected with 864 conductors. The flux per pole is 0.05 Wb. Calculate (i), the speed and (ii) the gross torque developed by the armature.

**Solution:**

$$I_a = 110 \text{ A}$$

$$E_b = V - I_a R_a = 480 - (110 \times 0.2) = 458 \text{ V}$$

$$E_b = \frac{\Phi Z N P}{60 A} \implies 458 = \frac{0.05 \times 864 \times N}{60} \left(\frac{6}{6}\right) \implies \therefore N = 636 \text{ r.p.m.}$$

$$T_a = 0.159 \Phi Z I_a \times (P/A) = 0.159 \times 0.05 \times 864 \times 110 (6/6) = 756.3 \text{ N-m}$$

Or

$$T_a = \frac{P_m}{\omega} = \frac{E_b I_a}{\frac{2\pi N}{60}} = 9.55 \frac{E_b I_a}{N} = 9.55 \times \frac{458 \times 110}{636} = 756.5 \text{ N-m}$$

**Example 6:** A 250-V, 4-pole, wave-wound d.c. series motor has 782 conductors on its armature. It has armature and series field resistance of 0.75 ohm. The motor takes a current of 40 A. Estimate its speed and gross torque developed if it has a flux per pole of 25 mWb.

**Solution:**

$$E_b = V - I_L R_a = 250 - (40 \times 0.75) = 220 \text{ V}$$

$$E_b = \frac{\Phi Z N P}{60 A} \implies 220 = \frac{25 \times 10^{-3} \times 782 \times N}{60} \left(\frac{4}{2}\right) \implies N = 337.6 \text{ r.p.m.}$$

$$T_a = 0.159 \Phi Z I_a \times (P/A) = 0.159 \times 25 \times 10^{-3} \times 782 \times 40 \times (4/2) = 249 \text{ N-m}$$

**Example 7:** A d.c. shunt machine develops an a.c. e.m.f. of 250 V at 1500 r.p.m. Find its torque and mechanical power developed for an armature current of 50 A. State the simplifying assumptions.

**Solution:**

**Note:** a.c. e.m.f. means induced voltage in armature, because the induced voltage in armature is a.c.

A given d.c. machine develops the same e.m.f. in its armature conductors whether running as a generator or as a motor. Only difference is that this armature e.m.f. is known as back e.m.f. when the machine is running as a motor.

$$\text{Mechanical power developed in the arm} = E_b I_a = 250 \times 50 = 12,500 \text{ W}$$

$$T_a = 9.55 E_b I_a / N = 9.55 \times 250 \times (50/1500) = \mathbf{79.6 \text{ N-m.}}$$



**Example 8:** Determine developed torque and shaft torque of 220-V, 4-pole series motor with 800 conductors wave-connected supplying a load of 8.2 kW by taking 45 A from the mains. The flux per pole is 25 mWb and its armature circuit resistance is 0.6 Ω.

**Solution:**

Developed torque or gross torque is the same thing as armature torque.

$$\therefore T_a = 0.159 \Phi Z I_a (P/A) = 0.159 \times 25 \times 10^{-3} \times 800 \times 45 (4/2) = 286.2 \text{ N-m}$$

$$E_b = V - I_a(R_a + R_{se}) = 220 - 45 \times (0.6 + 0) = 193 \text{ V}$$

Note:  $R_{se}$  is negligible because its value is small and not given .

$$\text{Now, } E_b = \Phi Z N \left(\frac{P}{A}\right) \implies 193 = 25 \times 10^{-3} \times 800 \times N \times (4/2) \implies \therefore N = 4.825 \text{ r.p.s.}$$

$$T_{sh} = \frac{P_{out}}{2\pi N} = \frac{8200}{2\pi \times 4.825} = 270.5 \text{ N-m}$$

**Example 9:** A 220-V d.c. shunt motor runs at 500 r.p.m. when the armature current is 50 A. Calculate the speed if the torque is doubled. Given that  $R_a = 0.2 \Omega$ .

**Solution:**

$T_a \propto \Phi I_a$ . Since  $\Phi$  is constant in shunt motor,  $T_a \propto I_a$

$$\therefore T_{a1} \propto I_{a1} \quad \dots(1)$$

$$T_{a2} \propto I_{a2} \quad \dots(2)$$

Divide equation (2) on equation (1), we get;

$$\therefore T_{a2}/T_{a1} = I_{a2}/I_{a1}$$

$$\text{As torque is doubled, } T_{a2}=2 T_{a1} \implies \boxed{T_{a2} / T_{a1} = 2}$$

$$\therefore 2 = I_{a2}/50 \text{ or } I_{a2} = 100 \text{ A}$$

$$E_{b1} = V - I_{a1}R_a = 220 - (50 \times 0.2) = 210 \text{ V}, \quad E_{b2} = V - I_{a2}R_a = 220 - (100 \times 0.2) = 200 \text{ V}$$

$$E_b = \frac{\Phi Z N P}{60 A} \implies E_b \propto N, \text{ as } \left(\frac{\Phi Z P}{60 A}\right) \text{ is constant.}$$

$$E_{b1} \propto N_1 \quad \dots(3)$$

$$E_{b2} \propto N_2 \quad \dots(4)$$

Divide equation (4) on equation (3), we get;

Now,  $E_{b2}/E_{b1} = N_2/N_1$  - since  $\Phi$  remains constant for shunt motor.

$$\therefore 200/210 = N_2/500 \implies \therefore N_2 = \mathbf{476 \text{ r.p.m.}}$$

**Example 10:** A 500-V, 37.3 kW, 1000 r.p.m. d.c. shunt motor has on full-load an efficiency of 90%. Determine (i) full-load line current (ii) full load shaft torque in N-m.

**Solution:**

(i)  $P_{output} = 37.3 \text{ kW}$

$$\text{Motor input} = 37,300/0.9 = 41,444 \text{ W, F.L. line current} = 41,444/500 = \mathbf{82.9 \text{ A}}$$

$$(ii) T_{sh} = 9.55 \times \frac{P_{output}}{N} = 9.55 \times \frac{37300}{1000} = 356 \text{ N-m}$$



**Example 11:** A 4-pole, 220-V shunt motor has 540 lap-wound conductor. It takes 32 A from the supply mains and develops output power of 5.595 kW. The field winding takes 1 A. The armature resistance is 0.09  $\Omega$  and the flux per pole is 30 mWb. Calculate (i) the speed and (ii) the shaft torque in newton-metre.

**Solution:**

$$I_a = I_L - I_{sh} = 32 - 1 = 31 \text{ A}; E_b = V - I_a R_a = 220 - (0.09 \times 31) = 217.2 \text{ V}$$

$$\text{Now, } E_b = \frac{\Phi Z N P}{60 A}$$

$$\therefore 217.2 = \frac{30 \times 10^{-3} \times 540 \times N \times 4}{60 \times 4}$$

(i)  $\therefore N = 804.4 \text{ r.p.m.}$

(ii)  $T_{sh} = 9.55 \times \frac{P_{\text{output}}}{N} = 9.55 \times \frac{5595}{804.4} = 66.5 \text{ N-m}$

**Example 12:** Find the no-load and full-load speeds for a four-pole, 220-V, and 20-kW, shunt motor having the following data :

Field-current = 5 amp, armature resistance = 0.04 ohm,

Flux per pole = 0.04 Wb, number of armature-conductors = 160, Two-circuit (wave-connection), full load current = 95 amp, No load current = 9 A. Neglect armature reaction.

**Solution:**

For shunt motor, ( $I_{sh}$ ) is constant for both cases.

**At no-load:**

$$I_{(\text{no-load})} = 9 \text{ A}, I_{sh} = 5 \text{ A} \quad \Longrightarrow \quad I_a = I_L - I_{sh} = 9 - 5 = 4 \text{ A}$$

$$E_{bo} = V - I_a R_a = 220 - 4 \times 0.04 = 219.84 \text{ volts}$$

$$E_{bo} = \frac{\Phi Z N_o \left(\frac{P}{A}\right)}{60} \quad \Longrightarrow \quad 219.84 = \frac{0.04 \times 160 \times N_o \left(\frac{4}{2}\right)}{60}$$

$\therefore$  No-Load speed,  $N_o = 1030.5 \text{ r.p.m.}$

**At full-load:**

$$I_{(\text{full-load})} = 95 \text{ A}, I_{sh} = 5 \text{ A} \quad \Longrightarrow \quad I_a = I_L - I_{sh} = 95 - 5 = 90 \text{ A}$$

$$E_b = V - I_a R_a = 220 - 90 \times 0.04 = 216.4 \text{ volts}$$

$$E_b = \frac{\Phi Z N \left(\frac{P}{A}\right)}{60}$$

$$216.4 = \frac{0.04 \times 160 \times N_{\text{full-load}} \left(\frac{4}{2}\right)}{60}$$

$\therefore$  Full-load speed,  $N_{\text{full-load}} = 1014.4 \text{ r.p.m.}$

Or,  $\frac{E_b}{E_{bo}} = \frac{N_{\text{full-load}}}{N_o} \quad \Longrightarrow \quad N_{\text{full-load}} = N_o \times \frac{E_b}{E_{bo}} = 1030.5 \times \frac{216.4}{219.84} = 1014.4 \text{ r.p.m}$

**Example 13:** Armature of a 6-pole, 6-circuit D.C. shunt motor takes 400 A at a speed of 350 r.p.m. The flux per pole is 80 milli-webers, the number of armature turns is 600, and 3% of the torque is lost in windage, friction and iron-loss. Calculate the brake-horse-power.

**Solution:**

**Note:**

- 1) 6-circuit, means lap wound armature with 6-parallel paths & 6-poles).
- 2) Brake horse power (B.H.P.) means output power in horse power.

$$P = A = 6 \text{ (lap wound)}$$

$$Z = 2 \times \text{No. of turns} = 2 \times 600 = 1200$$

$$E_b = \frac{\Phi Z N}{60} \left( \frac{P}{A} \right) = \frac{80 \times 10^{-3} \times 1200 \times 350}{60} \left( \frac{6}{6} \right) = 560 \text{ V}$$

$$I_a = I_L - I_{sh}$$

$$= 400 - 0 = 400 \text{ A, (} I_{sh} \text{ is negligible because its value ( or } R_{sh} \text{ value) is not given)}$$

$$\text{Gross mechanical power develop in armature (} P_m \text{)} = E_b I_a = 560 \times 400 = 224 \text{ Kw}$$

$$\therefore T_a = 9.55 \frac{E_b I_a}{N} = 9.55 \frac{P_m}{N} = 9.55 \frac{224000}{350} = 6112 \text{ N-m}$$

$$T_f = 0.03 T_a = 0.03 \times 6112 = 183.36 \text{ N-m}$$

$$T_a = T_f + T_{sh}$$

$$\therefore T_{sh} = T_a - T_f = 6112 - 183.36 = 5928.64 \text{ N-m}$$

$$T_{sh} = \frac{P_{out}}{\omega} = \frac{P_{out}}{2\pi \left( \frac{N}{60} \right)} \text{ N-m} \quad \text{where (} P_{out} \text{ in watt and N in r.p.m.)}$$

$$\therefore P_{out} = T_{sh} \times \omega = T_{sh} \times 2\pi \left( \frac{N}{60} \right) = 5928.64 \times 2\pi \left( \frac{350}{60} \right) = 217.186 \text{ kW}$$

**Note:** 1 HP. (horse power) = 746 watt

$$\text{Converting } P_{out} \text{ to BHP} = \frac{P_{out} \text{ in (watt)}}{746} = \frac{217186}{746} = 291.13 \text{ HP}$$

**Example 14:** Determine the torque established by the armature of a four-pole D.C. motor having 774 conductors, two paths in parallel, 24 milli-webers of pole-flux and the armature current is 50 Amps.

**Solution:**

Expression for torque in terms of the parameters concerned in this problem is as follows :

$$T_a = 0.159 \Phi Z I_a \left( \frac{P}{A} \right) \text{ N - m}$$

$$T_a = 0.159 \times (24 \times 10^{-3}) \times 774 \times 50 \times (4/2) = 295.36 \text{ Nw-m}$$



**Example 15:** A 500-V D.C. shunt motor draws a line-current of 5 A on light-load. If armature resistance is 0.15 ohm and field resistance is 200 ohms, determine the efficiency of the machine running as a generator delivering a load current of 40 Amps.

**Solution:**

**Note:** light load means approximately no-load condition ( $P_{out} = 0$ ).

**(i) No Load, running as a motor :**

Input Power =  $V \cdot I_{no-load} = 500 \times 5 = 2500$  watts ( as motor work on no-load ( $P_{out} = 0$ ), hence input power represent losses only ( $W_{No-load} = W_{cu-total at no-load} + W_{stray}$ )

$$I_{sh} = 500/200 = 2.5 \text{ A}, \quad I_a = I_L - I_{sh} = 5 - 2.5 = 2.5 \text{ A}$$

$$\text{Shunt cu loss} = I_{sh}^2 R_{sh} = (2.5)^2 \times 200 = 1250 \text{ watts}$$

Or

$$\text{Field cu loss} = V I_{sh} = 500 \times 2.5 = 1250 \text{ watts}$$

$$\text{Armature cu loss} = I_a^2 R_a = (2.5)^2 \times 0.15 = 0.94 \text{ watts} \cong 1 \text{ watts}$$

**Note:** (at no-load-armature cu loss is very small because ( $I_a$ ) at no load is very small as compared with ( $I_a$ ) at full load).

$$W_{total \text{ cu-loss at no-load}} = [\text{shunt cu loss} + \text{armature cu loss}] \\ = 1250 + 1 = 1251 \text{ watts}$$

$$W_{stray} = W_{No-load} - W_{cu-total at no-load} = 2500 - 1251 = 1249 \text{ watts}$$

**(ii) As a Generator, delivering 40 A to load :**

$$P_{out} = V \cdot I_L = 500 \times 40 = 20000 \text{ watts}$$

$$I_{sh} = 500/200 = 2.5 \text{ A}$$

$$I_a = I_L + I_{sh} = 40 + 2.5 = 42.5 \text{ A}$$

Losses :

$$\text{Field cu-loss} = I_{sh}^2 R_{sh} = (2.5)^2 \times 200 = 1250 \text{ watts}$$

$$\text{Armature cu-loss} = I_a^2 R_a = 42.5^2 \times 0.15 = 271 \text{ watts}$$

$$W_{total \text{ cu-loss at full-load}} = [\text{shunt cu loss} + \text{armature cu loss}] = 1250 + 271 = 1521 \text{ watts}$$

$$W_{total \text{ loss}} = W_{total \text{ cu-loss at full-load}} + W_{stray} = 1521 + 1249 = 2770 \text{ watts}$$

$$P_{input} = P_{out} + W_{total \text{ loss}} = 20000 + 2770 = 22770 \text{ watts}$$

$$\text{Generator Efficiency} = (20000/22770) \times 100 \% = 87.83 \%$$

**Alternative solution:**

**(i) No Load, running as a motor :**

$$I_{sh} = 500/200 = 2.5 \text{ A}$$

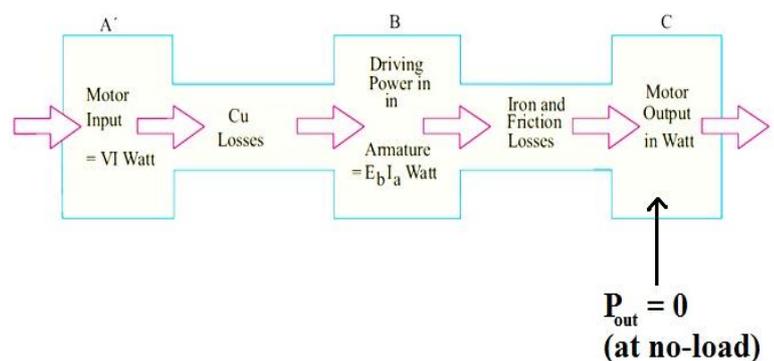
$$I_a = I_L - I_{sh} = 5 - 2.5 = 2.5 \text{ A}$$

$$E_b = V - I_a R_a = 500 - 2.5 \times 0.15 \\ = 499.625 \text{ V}$$

$$P_m = E_b I_a = 499.625 \times 2.5 = 1249 \text{ watts}$$

$$P_m = P_{out} + W_{stray} = 0 + W_{stray} \\ \text{(at no-load } P_{out} = 0)$$

$$\therefore W_{stray} = P_m = 1249 \text{ watts}$$



**Figure (14)**



**(ii) As a Generator, delivering 40 A to load :**

$$I_{sh} = 500/200 = 2.5 \text{ A}$$

$$I_a = I_L + I_{sh} = 40 + 2.5 = 42.5 \text{ A}$$

$$E_g = V + I_a R_a = 500 + 42.5 \times 0.15 = 506.375 \text{ V}$$

$$P_g = E_g I_a = 506.375 \times 42.5 = 21521 \text{ watts}$$

$$P_{input} = P_g + W_{stray} = 21521 + 1249 = 22770 \text{ watts}$$

$$P_{out} = V \cdot I_L = 500 \times 40 = 20000 \text{ watts}$$

$$\text{Generator Efficiency} = (20000/22770) \times 100 \% = 87.83 \%$$

**Extension to the Question :** At what speed should the Generator be run, if the shunt-field is not changed, in the above case ? Assume that the motor was running at 600 r.p.m. Neglect armature reaction.

**Solution.** As a motor on no-load,

$$E_{b0} = 500 - I_a R_a = 500 - 0.15 \times 2.5 = 499.625 \text{ V}$$

As a Generator with an armature current of 42.5 A,

$$E_g = 500 + 42.5 \times 0.15 = 506.375 \text{ V}$$

Since, the terminal voltage is same in both the cases, shunt field current remains as 2.5 amp. The e.m.f. then becomes proportional to the speed.

$$N = (506.375/499.625) \times 600 = 608.1 \text{ r.p.m.}$$

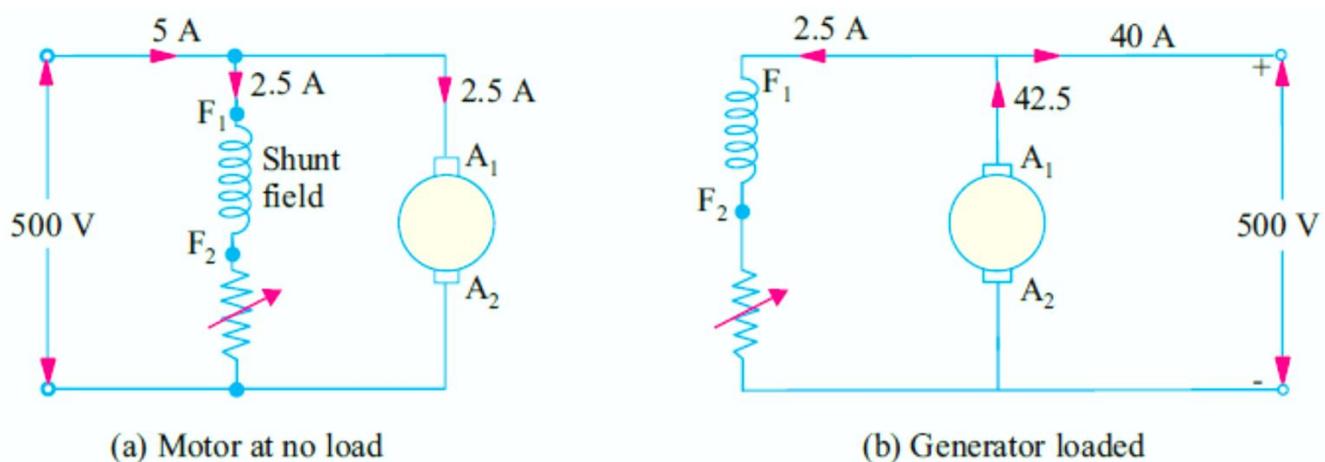


Figure (15)



**Example 16:** A d.c. series motor takes 40 A at 220 V and runs at 800 r.p.m. If the armature and field resistance are 0.2  $\Omega$  and 0.1  $\Omega$  respectively and the iron and friction losses are 0.5 kW, find the torque developed in the armature. What will be the output of the motor ?

**Solution:**

$$T_a = 9.55 \frac{E_b I_a}{N}$$

$$E_b = V - I_a(R_a + R_{se}) = 220 - 40(0.2 + 0.1) = 208 \text{ V}$$

$$\therefore T_a = 9.55 \times \left( \frac{208 \times 40}{800} \right) = \mathbf{99.3 \text{ N-m}}$$

Cu loss in armature and series-field resistance =  $40^2 \times 0.3 = 480 \text{ W}$

Iron and friction losses = 500 W

$\therefore$  Total losses = 480 + 500 = 980 W

Motor power input =  $V.I_L = 220 \times 40 = 8,800 \text{ W}$

Motor output ( $P_{out}$ ) =  $P_{input} - \text{Total losses} = 8,800 - 980 = 7,820 \text{ W} = \mathbf{7.82 \text{ kW}}$

**Example 17:** The armature winding of a 200-V, 4-pole, series motor is lap-connected. There are 280 slots and each slot has 4 conductors. The current is 45 A and the flux per pole is 18 mWb. The field resistance is 0.3  $\Omega$ ; the armature resistance 0.5  $\Omega$  and the iron and friction losses total 800 W. The pulley diameter is 0.41 m. Find the pull in newton at the rim of the pulley.

**Solution:**

Total number of conductors ( $Z$ ) =  $280 \times 4 = 1120$  conductors

$$E_b = V - I_a R_a = 200 - 45(0.5 + 0.3) = 164 \text{ V}$$

$$E_b = \frac{\Phi Z N}{60} \left( \frac{P}{A} \right) \quad \longrightarrow \quad 164 = \frac{18 \times 10^{-3} \times 1120 \times N}{60} \left( \frac{4}{4} \right)$$

$$\therefore N = \mathbf{488 \text{ r.p.m.}}$$

Total input ( $P_{input}$ ) =  $V.I_L = 200 \times 45 = 9,000 \text{ W}$

Cu loss =  $I_a^2(R_a + R_{se}) = 45^2 \times (0.3 + 0.5) = 1,620 \text{ W}$

Iron + Friction losses = 800 W

Total losses = 1,620 + 800 = 2,420 W

Output =  $P_{input} - \text{Total losses} = 9,000 - 2,420 = 6,580 \text{ W}$

$$\therefore T_{sh} = 9.55 \frac{P_{out}}{N} = 9.55 \times \frac{6580}{488} = 128 \text{ N-m}$$

Let  $F$  be the pull in newtons at the rim of the pulley.

$$\text{Then } T_{sh} = F \times r \quad \longrightarrow \quad 128.8 = F \times 0.205, \text{ where } r = \frac{D}{2} = \frac{0.41}{2} = 0.205 \text{ m}$$

$$\therefore F = 128.8 / 0.205 \text{ N} = \mathbf{628.29 \text{ N}}$$



**Example 18:** A 4-pole, 240 V, wave connected shunt motor gives 11.19 kW when running at 1000 r.p.m. and drawing armature and field currents of 50 A and 1.0 A respectively. It has 540 conductors. Its resistance is  $0.1 \Omega$ . Assuming a drop of 1 volt per brush, find (a) total torque (b) useful torque (c) useful flux / pole (d) rotational losses and (e) efficiency.

**Solution:**

$$E_b = V - I_a R_a - \text{brush drop} = 240 - (50 \times 0.1) - (2 \times 1) = 233 \text{ V}$$

Also  $I_a = 50 \text{ A}$ ,  $I_L = 50 + 1 = 51 \text{ A}$

(a) Armature torque  $T_a = 9.55 \frac{E_b I_a}{N} \text{ N-m} = 9.55 \times \left( \frac{233 \times 50}{1000} \right) = \mathbf{111 \text{ N-m}}$

(b)  $T_{sh} = 9.55 \frac{P_{out}}{N} \text{ N-m} = 9.55 \times \left( \frac{11190}{1000} \right) = \mathbf{106.9 \text{ N-m}}$

(c)  $E_b = \frac{\Phi Z N}{60} \left( \frac{P}{A} \right) \implies 233 = \frac{\Phi \times 540 \times 1000}{60} \left( \frac{4}{2} \right), \therefore \Phi = \mathbf{12.9 \text{ mWb}}$

(d) Armature input =  $V \cdot I_a = 240 \times 50 = \mathbf{12,000 \text{ W}}$

Armature Cu loss =  $I_a^2 R_a = 50^2 \times 0.1 = 250 \text{ W}$

Brush contact loss =  $I_a \cdot V_{\text{brush total}} = 50 \times 2 = 100 \text{ W}$

$\therefore$  Power developed =  $12,000 - 350 = 11,650 \text{ W}$

Output =  $11.19 \text{ kW} = 11,190 \text{ W}$

$\therefore$  Rotational losses ( $W_{\text{stray}}$ ) =  $11,650 - 11,190 = \mathbf{460 \text{ W}}$

**Or:**

$P_m = E_b I_a = 233 \times 50 = 11650 \text{ W}$

Rotational losses ( $W_{\text{stray}}$ ) =  $P_m - P_{out} = 11650 - 11190 = \mathbf{460 \text{ W}}$

**Or:**

Total losses =  $P_{input} - P_{output} = VI_L - P_{output} = 240 \times 51 - 11,190 = 1050 \text{ W}$

Armature Cu loss =  $I_a^2 R_a = 50^2 \times 0.1 = 250 \text{ W}$

Field cu loss =  $I_{sh} \cdot V = 1 \times 240 = 240 \text{ W}$

Brush contact loss =  $I_a \cdot V_{\text{brush total}} = 50 \times 2 = 100 \text{ W}$

$W_{\text{total cu loss}} = 250 + 240 + 100 = 590 \text{ W}$

Also, Total losses =  $W_{\text{total cu loss}} + W_{\text{stray}}$

$\therefore 1050 = 590 + W_{\text{stray}}$

$W_{\text{stray}} = 1050 - 590 = \mathbf{460 \text{ W}}$

**Note:** If the voltage drop across brushes is given, then we add power loss across brushes ( $= I_a V_{\text{total brush drop}}$ ) to total cu loss.

(e) Total motor input ( $P_{input}$ ) =  $VI_L = 240 \times 51 = 12,340 \text{ W}$

Motor output ( $P_{output}$ ) =  $11,190 \text{ W}$

$\therefore$  Efficiency =  $\frac{P_{out}}{P_{input}} \times 100\% = \frac{11190}{12240} \times 100\% = \mathbf{91.4\%}$



**Example 19:** A 460-V series motor runs at 500 r.p.m. taking a current of 40 A. Calculate the speed and percentage change in torque if the load is reduced so that the motor is taking 30 A. Total resistance of the armature and field circuits is  $0.8 \Omega$ . Assume flux is proportional to the field current.

**Solution:**

$$T \propto I_a \Phi$$
$$\Phi \propto I_{se} \propto I_a \quad (\text{for series motor})$$

$$\therefore T \propto I_a \Phi \propto (I_a)^2 \quad (\text{for series motor})$$

$$\therefore T_1 \propto 40^2 \quad \& \quad T_2 \propto 30^2$$

$$\therefore \frac{T_2}{T_1} = \frac{30^2}{40^2} = 0.5625$$

$$\therefore T_2 = 0.5625T_1$$

$$\begin{aligned} \therefore \text{Percentage change in torque is} &= \frac{T_1 - T_2}{T_1} \times 100 \\ &= \frac{T_1 - 0.5625T_1}{T_1} \times 100 = \frac{0.4375T_1}{T_1} \times 100 \\ &= \mathbf{43.75\%} \end{aligned}$$

$$\begin{aligned} \text{Now } E_{b1} &= 460 - (40 \times 0.8) = 428 \text{ V} \\ E_{b2} &= 460 - (30 \times 0.8) = 436 \text{ V} \end{aligned}$$

$$E_b = \frac{\Phi ZN}{60} \left( \frac{P}{A} \right), \text{ For series motor } \Phi \propto I_a$$

$$\therefore E_b \propto I_a N, \quad \text{As } \left( \frac{ZP}{60A} \right) \text{ is constant.}$$

$$\therefore N \propto \frac{E_b}{I_a}$$

$$N_1 \propto \frac{E_{b1}}{I_{a1}}, \quad \& \quad N_2 \propto \frac{E_{b2}}{I_{a2}}$$

$$\therefore \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}} \quad \longrightarrow \quad \frac{N_2}{500} = \frac{436}{428} \times \frac{40}{30}$$

$$\therefore N_2 = \mathbf{679 \text{ r.p.m.}}$$



**Example 20:** A 4-pole series motor has 944 wave-connected armature conductors. At a certain load, the flux per pole is 34.6 mWb and the total mechanical torque developed is 209 N-m. Calculate the line current taken by the motor and the speed at which it will run with an applied voltage of 500 V. Total motor resistance is 3 ohm.

**Solution:**

$$T_a = 0.159 \Phi Z I_a (P/A) \text{ N - m}$$

$$209 = 0.159 \times 34.6 \times 10^{-3} \times 944 \times I_a (4/2)$$

$$\therefore I_a = 20.1 \text{ A}$$

$$E_b = V - I_a R_a = 500 - 20.1 \times 3 = 439.7 \text{ V}$$

Now, speed may be found either by using the relation for  $E_b$  or  $T_a$ .

$$E_b = \Phi Z N \left( \frac{P}{A} \right) \quad \longrightarrow \quad 439.7 = 34.6 \times 10^{-3} \times 944 \times N \times \left( \frac{4}{2} \right)$$

$$\therefore N = 6.73 \text{ r.p.s. or } \mathbf{403.6 \text{ r.p.m.}}$$

**Example 21:** A 250-V shunt motor runs at 1000 r.p.m. at no-load and takes 8A. The total armature and shunt field resistances are respectively 0.2  $\Omega$  and 250  $\Omega$ . Calculate the speed when loaded and taking 50 A. Assume the flux to be constant.

**Solution:**

$$I_{sh} = 250/250 = 1 \text{ A}$$

$$E_{b0} = V - I_{a0} R_a = 250 - (7 \times 0.2) = 248.6 \text{ V}$$

$$E_b = V - I_a R_a = 250 - (49 \times 0.2) = 240.2 \text{ V}$$

$$\frac{E_b}{E_{b0}} = \frac{N}{N_0} \quad \longrightarrow \quad \frac{240.2}{248.6} = \frac{N}{1000}$$

$$\therefore N = \mathbf{966.2 \text{ r.p.m.}}$$

**Example 22:** A d.c. series motor operates at 800 r.p.m. with a line current of 100 A from 230-V mains. Its armature circuit resistance is 0.15  $\Omega$  and its field resistance 0.1  $\Omega$ . Find the speed at which the motor runs at a line current of 25 A, assuming that the flux at this current is 45 per cent of the flux at 100 A.

**Solution:**

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}, \quad \Phi_2 = 0.45 \Phi_1$$

$$E_{b1} = 230 - (0.15 + 0.1) \times 100 = 205 \text{ V}$$

$$E_{b2} = 230 - 25 \times 0.25 = 223.75 \text{ V}$$

$$\frac{N_2}{800} = \frac{223.75}{205} \times \frac{\Phi_1}{0.45 \Phi_1}$$

$$\therefore N_2 = \mathbf{1940 \text{ r.p.m.}}$$



**Example 23:** A 230-V d.c. shunt motor has an armature resistance of  $0.5 \Omega$  and field resistance of  $115 \Omega$ . At no load, the speed is 1,200 r.p.m. and the armature current 2.5 A. On application of rated load, the speed drops to 1,120 r.p.m. Determine the line current and power input when the motor delivers rated load.

**Solution:**

$$N_1 = 1200 \text{ r.p.m.}, \quad E_{b1} = 230 - (0.5 \times 2.5) = 228.75 \text{ V}$$

$$N_2 = 1120 \text{ r.p.m.}, \quad E_{b2} = 230 - 0.5 I_{a2}$$

$$\text{Now, } \frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1}$$

$$\therefore \frac{230 - 0.5 I_{a2}}{228.75} = \frac{1120}{1200}$$

$$\therefore I_{a2} = \mathbf{33 \text{ A}}$$

$$\text{Line current drawn by motor} = I_{a2} + I_{sh} = 33 + (230/115) = \mathbf{35 \text{ A}}$$

$$\text{Power input at rated load} = 230 \times 35 = \mathbf{8,050 \text{ W}}$$

**Example 24:** A belt-driven 100-kW, shunt generator running at 300 r.p.m. on 220-V bus bars continues to run as a motor when the belt breaks, then taking 10 kW. What will be its speed? Given armature resistance =  $0.025 \Omega$ , field resistance =  $60 \Omega$  and contact drop under each brush = 1 V, Ignore armature reaction.

**Solution:**

**As Generator** [Figure (16-a)]

$$\text{Load current, } I_L = 100,000/220 = 454.55 \text{ A};$$

$$I_{sh} = 220/60 = 3.67 \text{ A}$$

$$I_a = I_L + I_{sh} = 458.22 \text{ A}$$

$$E_{g1} = 220 + (458.2 \times 0.025) + (2 \times 1) = 233.45 \text{ V}$$

$$N_1 = 300 \text{ r.p.m.}$$

**As Motor** [Figure (16-b)]

$$\text{Input line current } (I_L) = 100,000/220 = 45.45 \text{ A}$$

$$I_{sh} = 220/60 = 3.67 \text{ A}$$

$$I_a = I_L - I_{sh} = 45.45 - 3.67 = 41.78 \text{ A}$$

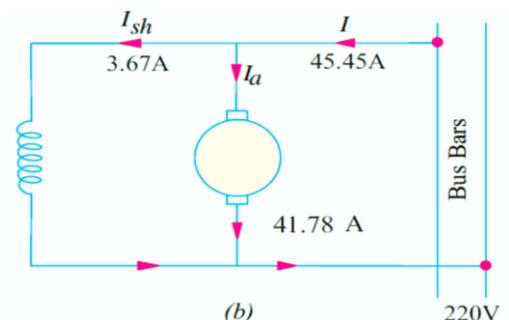
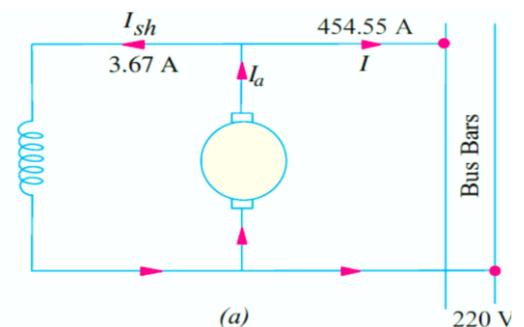
$$E_{b2} = 220 - (41.78 \times 0.025) - (2 \times 1) = 216.96 \text{ V}$$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{g1}} \times \frac{\Phi_1}{\Phi_2},$$

since  $\Phi_2 = \Phi_1$  for shunt machine because  $I_{sh}$  is constant.

$$\therefore \frac{N_2}{300} = \frac{216.96}{233.45}$$

$$\therefore N_2 = \mathbf{279 \text{ r.p.m.}}$$



**Figure (16)**



**Example 25:** A d.c. shunt machine generates 250-V on open circuit at 1000 r.p.m. Effective armature resistance is  $0.5 \Omega$ , field resistance is  $250 \Omega$ , input to machine running as a motor on no load is 4 A at 250 V. Calculate speed of machine as a motor taking 40 A at 250 V. Armature reaction weakens field by 4%.

**Solution:**

**When working as generator, generate 250 V on open circuit.**

$E_g = 250\text{V}$ ,  $N_g = 1000$  r.p.m.,  $\Phi_1 =$  flux of generator.

**When working as motor on no-load.**

Now,  $I_{sh} = 250/250 = 1$  A; Hence,  $I_{a0} = 4 - 1 = 3$  A,  $\Phi_2 =$  flux of motor on no-load

$$E_{b0} = 250 - 0.5 \times 3 = 248.5 \text{ V}$$

$$\frac{N_o}{N_g} = \frac{E_{b0}}{E_g} \times \frac{\Phi_1}{\Phi_o}, \quad \Phi_o = \Phi_1$$

$$\frac{N_o}{1000} = \frac{248.5}{250}$$

Hence,  $N_o = 994$  r.p.m.

**Note:**  $\Phi_o = \Phi_1$  (flux is constant whether the machine is working as generator or as motor on no-load, but it reduced when loaded motor as mentioned in example)

**When working as motor on-Load**

$$I_a = 40 - 1 = 39 \text{ A}$$

$$E_b = 250 - 39 \times 0.5 = 230.5 \text{ V}$$

$$\begin{aligned} \Phi_2 &= \Phi_o - 0.04\Phi_o \\ &= 0.96 \Phi_o \end{aligned}$$

$$\therefore \Phi_o/\Phi_2 = 1/0.96$$

$$\frac{N_2}{N_o} = \frac{E_b}{E_{b0}} \times \frac{\Phi_o}{\Phi_2}$$

$$\frac{N_2}{994} = \frac{230.5}{248.5} \times \frac{\Phi_o}{0.96\Phi_o}$$

$N_2 = 960.4$  r.p.m.

**Alternative solution:**

**When working as generator, generate 250 V on open circuit.**

$E_g = 250\text{V}$ ,  $N_g = 1000$  r.p.m.,  $\Phi_1 =$  flux of generator.



### When working as motor on-Load

$$I_a = 40 - 1 = 39 \text{ A}$$

$$E_b = 250 - 39 \times 0.5 = 230.5 \text{ V}$$

$$\begin{aligned} \Phi_2 &= \Phi_1 - 0.04\Phi_1 \\ &= 0.96 \Phi_1 \end{aligned}$$

$$\therefore \Phi_0/\Phi_2 = 1/0.96$$

$$\frac{N_2}{N_g} = \frac{E_b}{E_g} \times \frac{\Phi_1}{\Phi_2}$$

$$\frac{N_2}{1000} = \frac{230.5}{250} \times \frac{\Phi_1}{0.96\Phi_1}$$

$$N_2 = \mathbf{960.4 \text{ r.p.m.}}$$

**Example 26:** A 250-V shunt motor giving 14.92 kW at 1000 r.p.m. takes an armature current of 75 A. The armature resistance is 0.25 ohm and the load torque remains constant. If the flux is reduced by 20 percent of its normal value before the speed changes, find the instantaneous value of the armature current and the torque. Determine the final value of the armature current and speed.

### Solution:

$$E_{b1} = 250 - 75 \times 0.25 = 231.25 \text{ V, as in Figure (17).}$$

When flux is reduced by 20%, the back e.m.f. is also reduced instantly by 20% because speed remains constant (before speed changes) due to inertia of the heavy armature.

$$\Phi_2 = \Phi_1 - 0.2\Phi_1$$

$$\Phi_2 = 0.8\Phi_1$$

$$E_b = \Phi ZN \left( \frac{P}{A} \right) \longrightarrow \begin{matrix} E_b \propto \Phi N \\ E_b \propto \Phi \end{matrix} \text{ where } N \text{ is constant.}$$

$$\therefore \frac{(E_b)_{\text{inst}}}{E_{b1}} = \frac{\Phi_2}{\Phi_1} \longrightarrow \frac{(E_b)_{\text{inst}}}{231.25} = \frac{0.8\Phi_1}{\Phi_1}$$

$$\therefore \text{Instantaneous value of back e.m.f. } (E_b)_{\text{inst}} = 231.25 \times 0.8 = \mathbf{185 \text{ V}}$$

$$(I_a)_{\text{inst}} = \frac{V - (E_b)_{\text{inst}}}{R_a} = \frac{250 - 185}{0.25} = \mathbf{260 \text{ A}}$$

$$\begin{aligned} \text{Instantaneous value of the torque } (T_a)_{\text{inst}} &= 9.55 \frac{(E_b)_{\text{inst}} (I_a)_{\text{inst}}}{N} \text{ N-m} \\ &= 9.55 \times \frac{185 \times 260}{1000} = \mathbf{459 \text{ N-m}} \end{aligned}$$



### Steady Conditions

Since torque ( $T \propto \Phi I_a$ ) remains constant,  $\Phi_1 I_{a1} = \Phi_2 I_{a2}$

$$I_{a2} = (\Phi_1 I_{a1}) / \Phi_2 = 75 \times (\Phi_1 / 0.8 \Phi_1) = \mathbf{93.7 \text{ A}}$$

$$\therefore E_{b2} = 250 - 93.7 \times 0.25 = 226.6 \text{ V}$$

Now,

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

$$\frac{N_2}{1000} = \frac{226.6}{231.25} \times \frac{\Phi_1}{0.8 \Phi_1}$$

$$\therefore N_2 = \mathbf{1225 \text{ r.p.m.}}$$

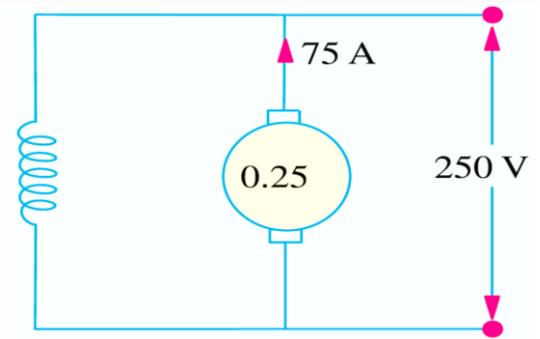


Figure (17)

**Example 27:** A 200-V, d.c. shunt motor takes 4 A at no-load when running at 700 r.p.m. The field resistance is 100  $\Omega$ . The resistance of armature at standstill gives a drop of 6 volts across armature terminals when 10 A were passed through it. Calculate (a) speed on load (b) torque in N-m and (c) efficiency. The normal input of the motor is 8 kW.

### Solution:

(a)  $I_{sh} = 200/100 = 2 \text{ A}$

F.L. Power input = 8,000 W

F.L. line current ( $I_L$ ) = 8,000/200 = 40 A

$I_a = 40 - 2 = 38 \text{ A}$ ;  $R_a = 6/10 = 0.6 \Omega$

$E_{b0} = 200 - 2 \times 0.6 = 198.8 \text{ V}$ ;

$E_b = 200 - 38 \times 0.6 = 177.2 \text{ V}$

Now,

$$\frac{N}{N_0} = \frac{E_b}{E_{b0}} \quad \longrightarrow \quad \frac{N}{700} = \frac{177.2}{198.8}$$

$N = \mathbf{623.9 \text{ r.p.m.}}$

(b)  $T_a = 9.55 (E_b I_a / N) = 9.55 \times (177.2 \times 38 / 623.9) = \mathbf{103 \text{ N-m}}$

(c) N.L. power input = 200  $\times$  4 = 800 W

N.L. Arm. Cu loss =  $I_a^2 R_a = 2^2 \times 0.6 = 2.4 \text{ W}$

Constant losses ( $W_{stray} + \text{shunt cu loss}$ ) = 800 - 2.4 = 797.6 W

F.L. arm. Cu loss = 38<sup>2</sup>  $\times$  0.6 = 866.4 W

Total F.L. losses = 797.6 + 866.4 = 1664 W

F.L. output ( $P_{out}$ ) =  $P_{input} - \text{total F.L. losses} = 8,000 - 1664 = 6336 \text{ W}$

F.L. Motor efficiency = 6336/8,000 = 0.792 or  $\mathbf{79.2 \%}$

Or:



$$\text{N.L. power input} = 200 \times 4 = 800 \text{ W}$$

$$\text{N.L. Arm. Cu loss} = I_a^2 R_a = 2^2 \times 0.6 = 2.4 \text{ W}$$

$$\text{N.L. Shunt Cu loss} = I_{sh} V = 2 \times 200 = 400 \text{ W}$$

$$\text{Total N.L. Cu loss} = 2.4 + 400 = 402.4 \text{ W}$$

$$W_{\text{stray}} = \text{N.L. power input} - \text{Total N.L. Cu loss} = 800 - 402.4 = 397.6 \text{ W}$$

$$\text{F.L. arm. Cu loss} = 382 \times 0.6 = 866.4 \text{ W}$$

$$\text{F.L. Shunt Cu loss} = I_{sh} V = 2 \times 200 = 400 \text{ W}$$

$$\text{Total F.L. Cu loss losses} = 866.4 + 400 = 1266.4 \text{ W}$$

$$\begin{aligned} \text{Total motor loss} &= \text{Total F.L. Cu loss losses} + W_{\text{stray}} \\ &= 1266.4 + 397.6 = 1664 \text{ W} \end{aligned}$$

$$\text{F.L. output } (P_{\text{out}}) = P_{\text{input}} - \text{total F.L. losses} = 8,000 - 1664 = 6336 \text{ W}$$

$$\text{F.L. Motor efficiency} = 6336/8,000 = 0.792 \text{ or } \mathbf{79.2 \%}$$

**Example 28:** The input to 220-V, d.c. shunt motor is 11kW. Calculate (a) the torque developed (b) the efficiency (c) the speed at this load. The particulars of the motor are as follows :

No-load current = 5 A; No-load speed = 1150 r.p.m.

Arm. resistance = 0.5  $\Omega$ ; shunt field resistance = 110  $\Omega$ .

**Solution:**

$$\text{N.L. input} = 220 \times 5 = 1,100 \text{ W}$$

$$I_{sh} = 220/110 = 2 \text{ A}, I_{a0} = 5 - 2 = 3 \text{ A}$$

$$\text{No-load armature Cu loss} = 3^2 \times 0.5 = 4.5 \text{ W}$$

$$\therefore \text{Constant losses } (W_{\text{stray}} + \text{shunt cu loss}) = 1,100 - 4.5 = 1,095.5 \text{ W}$$

**When input is 11 kW.**

$$\text{Input current} = 11,000/220 = 50 \text{ A}$$

$$\text{Armature current } (I_a) = 50 - 2 = \mathbf{48 \text{ A}}$$

$$\text{F.L. Arm. Cu loss} = 48^2 \times 0.5 = 1,152 \text{ W};$$

$$\text{F.L. total loss} = \text{F.L. Arm. Cu loss} + \text{Constant losses} = 1152 + 1095.5 = 2248 \text{ W}$$

$$P_{\text{output}} = P_{\text{input}} - \text{F.L. total loss} = 11,000 - 2,248 = 8,752 \text{ W}$$

$$\mathbf{(b) \text{ Efficiency} = 8,752 \times 100/11,000 = 79.6\%}$$

$$\mathbf{(c) \text{ Back e.m.f. at no-load } (E_{b0}) = 220 - (3 \times 0.5) = 218.5 \text{ V}}$$

$$\text{Back e.m.f. at given load } (E_b) = 220 - (48 \times 0.5) = 196 \text{ V}$$

$$\frac{N_{\text{F.L.}}}{N_0} = \frac{E_b}{E_{b0}} \quad \longrightarrow \quad \therefore \text{Speed } N_{\text{F.L.}} = 1,150 \times (196/218.5) = \mathbf{1,031 \text{ r.p.m.}}$$

$$\mathbf{(a) T_a = 9.55 \times \left( \frac{196 \times 48}{1031} \right) = 87.1 \text{ N-m}}$$



**Example 29:** The armature circuit resistance of a 18.65 kW 250-V series motor is 0.1 Ω, the brush voltage drop is 3V, and the series field resistance is 0.05. When the motor takes 80 A, speed is 600 r.p.m. Calculate the speed when the current is 100 A

**Solution:**

$$E_{b1} = 250 - 80(0.1 + 0.05) - 3 = 235 \text{ V.}$$

$$E_{b2} = 250 - 100(0.1 + 0.05) - 3 = 232 \text{ V}$$

Since  $\Phi \propto I_a$ ,

hence,  $\Phi_1 \propto 80$ ,  $\Phi_2 \propto 100$ ,  $(\Phi_1/\Phi_2) = (80/100)$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

$$\frac{N_2}{600} = \frac{232}{235} \times \frac{80}{100}$$

$$\therefore N_2 = 474 \text{ r.p.m.}$$

**Example 30:** A 220-volt d.c. series motor is running at a speed of 800 r.p.m. and draws 100 A. Calculate at what speed the motor will run when developing half the torque. Total resistance of the armature and field is 0.1 ohm. Assume that the magnetic circuit is unsaturated.

**Solution:**

**Note:** magnetic circuit unsaturated means, ( $\Phi$  is not constant)  $\Phi \propto I_a$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}}, (\Phi \propto I_a)$$

Since field is unsaturated,  $T_a \propto \Phi I_a \propto I_a^2$ ,  $(\therefore T_1 \propto I_{a1}^2 \text{ and } T_2 \propto I_{a2}^2)$

or

$$\frac{T_2}{T_1} = \left(\frac{I_{a2}}{I_{a1}}\right)^2$$

$$\frac{1}{2} = \left(\frac{I_{a2}}{I_{a1}}\right)^2$$

$$\therefore I_{a2} = \frac{I_{a1}}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.7 \text{ A}$$

$$E_{b1} = 220 - 100 \times 0.1 = 210 \text{ V}$$

$$E_{b2} = 220 - 0.1 \times 70.7 = 212.9 \text{ V}$$

$$\therefore \frac{N_2}{800} = \frac{212.9}{210} \times \frac{100}{70.7}$$

$$\therefore N_2 = 1147 \text{ r.p.m.}$$



**Example 31:** A 4-pole d.c. motor runs at 600 r.p.m. on full load taking 25 A at 450 V. The armature is lap-wound with 500 conductors and flux per pole is expressed by the relation.

$$\Phi = (1.7 \times 10^{-2} \times I^{0.5}) \text{ weber}$$

where (I) is the motor current. If supply voltage and torque are both halved, calculate the speed at which the motor will run. Ignore stray losses.

**Solution:**

Let us first find  $R_a$ .

Now

$$E_b = \frac{\Phi Z N \left(\frac{P}{A}\right)}{60} = \frac{1.7 \times 10^{-2} \times \sqrt{I} \times 500 \times 600}{60} \left(\frac{4}{4}\right) = \frac{1.7 \times 10^{-2} \times \sqrt{25} \times 500 \times 600}{60} \left(\frac{4}{4}\right) = 425 \text{ V}$$

$$E_b = V - I_a R_a$$

$$I_a R_a = V - E_b = 450 - 425 = 25 \text{ V}$$

$$\therefore R_a = 25/25 = 1.0 \Omega$$

**Now in the 1st Case**

$$T_1 \propto \Phi_1 I_1$$

$$\therefore T_1 \propto 1.7 \times 10^{-2} \times \sqrt{25} \times 25$$

Similarly

$$T_2 \propto 1.7 \times 10^{-2} \times \sqrt{I} \times I = 1.7 \times 10^{-2} \times I^{\left(\frac{3}{2}\right)}$$

$$; \text{ Now } T_1 = 2T_2$$

$$\therefore (1.7 \times 10^{-2} \times \sqrt{25} \times 25) = 2 \times (1.7 \times 10^{-2} \times I^{\left(\frac{3}{2}\right)})$$

$$\therefore I = (125/2)^{2/3} = 15.75 \text{ A}$$

$$E_{b1} = 425 \text{ V}$$

$$E_{b2} = 225 - (15.75 \times 1) = 209.3 \text{ V}$$

$$\text{Using the relation } \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

$$\frac{N_2}{600} = \frac{209.3}{425} \times \frac{1.7 \times 10^{-2} \times 5}{1.7 \times 10^{-2} \times \sqrt{15.75}}$$

$$N_2 = \mathbf{372 \text{ r.p.m.}}$$



## Tutorial Problems (2)

[1] Calculate the torque in newton-metre developed by a 440-V d.c. motor having an armature resistance of  $0.25 \Omega$  and running at 750 r.p.m. when taking a current of 60 A. **[325 N-m]**

[2] A 4-pole, lap-connected d.c. motor has 576 conductors and draws an armature current of 10 A. If the flux per pole is 0.02 Wb, calculate the armature torque developed. **[18.3 N-m]**

[3] (a) A d.c. shunt machine has armature and field resistances of  $0.025 \Omega$  and  $80 \Omega$  respectively. When connected to constant 400-V bus-bars and driven as a generator at 450 r.p.m., it delivers 120 kW. Calculate its speed when running as a motor and absorbing 120 kW from the same bus-bars.

(b) Deduce the direction of rotation of this machine when it is working as a motor assuming a clockwise rotation as a generator. **[(a) 435 r.p.m. (b) Clockwise]**

[4] The armature current of a series motor is 60 A when on full-load. If the load is adjusted to that this current decreases to 40-A, find the new torque expressed as a percentage of the full-load torque. The flux for a current of 40 A is 70% of that when current is 60 A. **[46.7%]**

[5] A 4-pole, d.c. shunt motor has a flux per pole of 0.04 Wb and the armature is lap-wound with 720 conductors. The shunt field resistance is  $240 \Omega$  and the armature resistance is  $0.2 \Omega$ . Brush contact drop is 1V per brush. Determine the speed of the machine when running (a) as a motor taking 60 A and (b) as a generator supplying 120 A. The terminal voltage in each case is 480 V. **[972 r.p.m. ; 1055 r.p.m.]**

[6] A 25-kW shunt generator is delivering full output to 400-V bus-bars and is driven at 950 r.p.m. by belt drive. The belt breaks suddenly but the machine continues to run as a motor taking 25 kW from the bus-bars.

At what speed does it run ? Take armature resistance including brush contact resistance as  $0.5 \Omega$  and field resistance as  $160 \Omega$ . **[812.7 r.p.m.]**

[7] A 4-pole, d.c. shunt motor has a wave-wound armature with 65 slots each containing 6 conductors. The flux per pole is 20 mWb and the armature has a resistance of  $0.15 \Omega$ . Calculate the motor speed when the machine is operating from a 250-V supply and taking a current of 60 A. **[927 r.p.m.]**



[8] A 500-V, d.c. shunt motor has armature and field resistances of  $0.5 \Omega$  and  $200 \Omega$  respectively. When loaded and taking a total input of 25 kW, it runs at 400 r.p.m. Find the speed at which it must be driven as a shunt generator to supply a power output of 25 kW at a terminal voltage of 500 V. **[442 r.p.m.]**

[9] A d.c. shunt motor runs at 900 r.p.m. from a 400 V supply when taking an armature current of 25 A. Calculate the speed at which it will run from a 230 V supply when taking an armature current of 15 A. The resistance of the armature circuit is  $0.8 \Omega$ . Assume the flux per pole at 230 V to have decreased to 75% of its value at 400 V. **[688.4 r.p.m.]**

[10] A shunt machine connected to 250V mains has an armature resistance of  $0.12 \Omega$  and field resistance of  $100 \Omega$ . Find the ratio of the speed of the machine as a generator to the speed as a motor, if line current is 80 A in both cases. **[1.08]**

[11] A 20-kW d.c. shunt generator delivering rated output at 1000 r.p.m. has a terminal voltage of 500 V. The armature resistance is  $0.1 \Omega$ , voltage drop per brush is 1 volt and the field resistance is  $500 \Omega$ .

Calculate the speed at which the machine will run as a motor taking an input of 20 kW from a 500 V d.c. supply. **[976.1 r.p.m.]**

[12] A 4-pole, 250-V, d.c. shunt motor has a lap-connected armature with 960 conductors. The flux per pole is  $2 \times 10^{-2}$  Wb. Calculate the torque developed by the armature and the useful torque in newton-metre when the current taken by the motor is 30A. The armature resistance is  $0.12 \Omega$  and the field resistance is  $125 \Omega$ . The rotational losses amount to 825 W. **[85.5 N-m ; 75.3 N-m]**



### 13. D.C. Motor Characteristics

The performance of a d.c motor under various conditions can be judged by the following characteristics.

i) *Torque - armature current characteristics ( $T$  Vs  $I_a$ ):* The graph showing the relationship between the torque and the armature current is called a torque-armature current characteristic. These are also called electrical characteristics.

ii) *Speed - armature current characteristics ( $N$  Vs  $I_a$ ):* The graph showing the relationship between the speed and armature current characteristics,

iii) *Speed - torque characteristics ( $N$  Vs  $T$ ):* The graph showing the relationship between the speed and the torque of the motor is called speed-torque characteristics of the motor. These are also called mechanical characteristics.

The nature of these characteristics can easily be obtained by using speed and torque equations. These characteristics play a very important role in selecting a type of motor for a particular application.

#### (a) Characteristics of D.C. Shunt Motor

i) *Torque - armature current characteristics ( $T$  Vs  $I_a$ )*

For a d.c. motor  $T \propto \Phi I_a$

For constant  $R_{sh}$  &  $V_{supply}$ ,  $I_{sh}$  is also constant and hence flux is also constant.

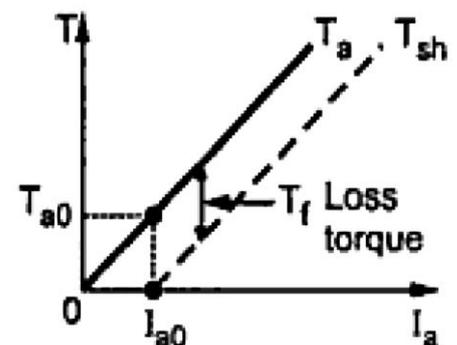
$$T \propto I_a$$

The equation represents a straight line, passing through the origin, as shown in the **Figure (18)**. Torque increases linearly with armature current. It is seen earlier that armature current is decided by the load. So as load increases, armature current increases, increasing the torque developed linearly.

Now if shaft torque is plotted against armature current, it is known that shaft torque is less than the armature torque and the difference between the two is loss torque  $T_f$  as shown.

On no load  $T_{sh} = 0$  but armature torque is present which is just enough to overcome stray losses shown as  $T_{a0}$ . The current required is  $I_{a0}$  on no load to produce  $T_{a0}$  and hence  $T_{sh}$  graph has an intercept of  $I_{a0}$  on the current axis.

To generate high starting torque, this type of motor requires a large value of armature current at start. This may damage the motor hence d.c. shunt motors can develop moderate starting torque and hence suitable for such applications where starting torque requirement is moderate.



**Figure (18):**  $T$  Vs  $I_a$  for shunt motor



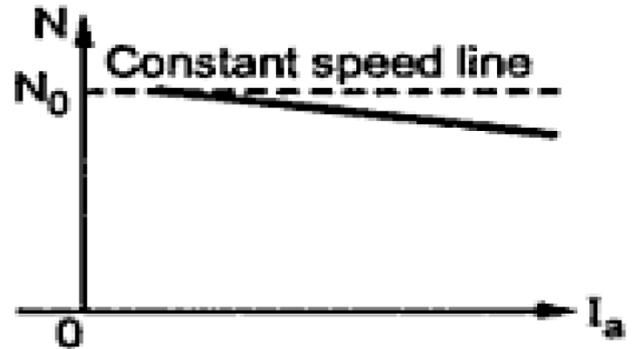
### ii) Speed - armature current characteristics ( $N$ Vs $I_a$ )

From the speed equation we get,

$$N \propto \frac{V - I_a R_a}{\phi}$$

$$\propto V - I_a R_a \quad \text{as } \phi \text{ is constant.}$$

So as load increases, the armature current increases and hence drop  $I_a R_a$  also increases.



**Figure (19):**  $N$  Vs  $I_a$  for shunt motor

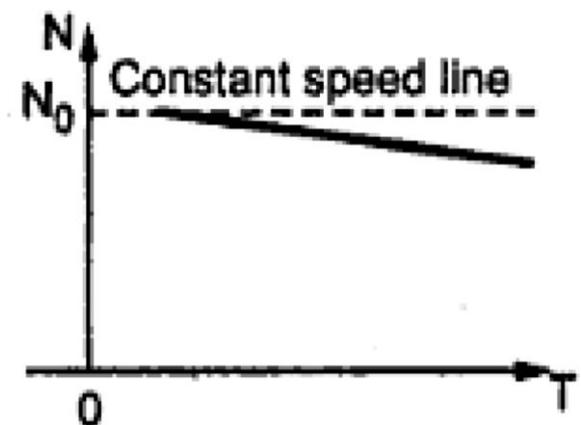
Hence for constant supply voltage,  $(V - I_a R_a)$  decreases and hence speed reduces. But as  $R_a$  is very small, for change in  $I_a$  from no load to full load, drop  $(I_a R_a)$  is very small and hence drop in speed is also not significant from no load to full load. So the characteristic is slightly drooping as shown in the **Figure (19)**. But for all practical purposes these types of motors are considered to be constant speed motors.

**Note:** shunt motors are used for applications that required constant speed.

### iii) Speed - torque characteristics ( $N$ Vs $T$ )

These characteristics can be derived from the above two characteristics.

This graph is similar to speed - armature current characteristics as torque is proportional to the armature current. This curve shows that the speed almost remains constant though torque changes from no load to full load conditions. This is shown in the **Figure (20)**.



**Figure (20):**  $N$  Vs  $T$  for shunt motor



## (b) Characteristics of D.C. Series Motor

### *i) Torque - armature current characteristics (T Vs I<sub>a</sub>)*

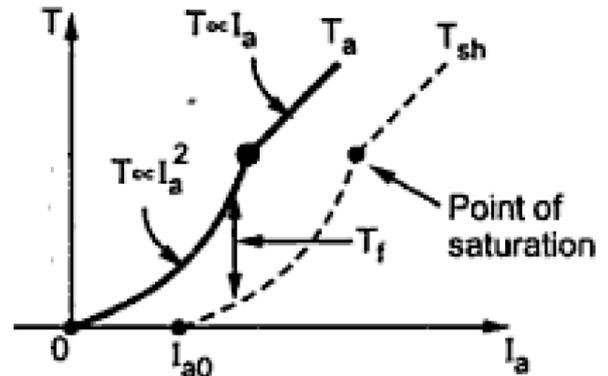
In case of series motor the series field winding is carrying the entire armature current. So flux produced is proportional to the armature current.

$$\Phi \propto I_a$$

Hence

$$T_a \propto \Phi I_a \propto (I_a)^2$$

Thus torque in case of series motor is proportional to the square of the armature current. This relation is parabolic in nature as shown in the **Figure (21)**.



**Figure (21):** T Vs I<sub>a</sub> for series motor

As load increases, armature current increases and torque produced increases proportional to the square of the armature current up to a certain limit.

As the entire (I<sub>a</sub>) passes through the series field, there is a property of an electromagnet called saturation, may occur. Saturation means though the current through the winding increases, the flux produced remains constant. Hence after saturation the characteristics take the shape of straight line as flux becomes constant, as shown. The difference between T<sub>a</sub> and T<sub>sh</sub> is loss torque T<sub>f</sub> which is also shown in the **Figure (21)**.

At start as  $T \propto (I_a)^2$ , these types of motors can produce high torque for small amount of armature current hence the series motors are suitable for the applications which demand high starting torque.

### *ii) Speed - armature current characteristics (N Vs I<sub>a</sub>)*

From the equation we get,  $N \propto \frac{E_b}{\Phi} \propto \frac{V - I_a R_a - I_a R_{se}}{I_a}$  as  $\Phi \propto I_a$  in case of series motor

Now the values of R<sub>a</sub> and R<sub>se</sub> are so small that the effect of change in I<sub>a</sub> on speed overrides the effect of change in  $V - I_a R_a - I_a R_{se}$  on the speed.

Hence in the speed equation, E<sub>b</sub> = V and can be assumed constant. So speed equation reduces to,

$$N \propto \frac{1}{I_a}$$

So speed - armature current characteristics is rectangular hyperbola type as shown in the **Figure (22)**.



**Figure (22):** N Vs I<sub>a</sub> for series motor



**Note:** When load current & hence ( $I_a$ ) falls to a small value, speed becomes dangerously high. Hence series motor should never be started without some mechanical (not belt-driven) load on it, otherwise it may develop excessive speed & get damaged due to heavy centrifugal forces.

It should be noted that, series motor is variable speed motor.

### iii) Speed - torque characteristics ( $N$ Vs $T$ )

In case of series motors,  $T \propto (I_a)^2$  and  $N \propto \frac{1}{I_a}$

Hence we can write,

$$N \propto \frac{1}{\sqrt{T}}$$

Thus as torque increases when load increases, the speed decreases. On no load, torque is very less and hence speed increases to dangerously high value. Thus the nature of the speed-torque characteristics is similar to the nature of the speed-armature current characteristics.



**Figure (23):**  $N$  Vs  $T$  for series motor

The speed-torque characteristics of a series motor is shown in the **Figure (23)**.

### Why Series Motor is Never Started on No Load?

It is seen earlier that motor armature current is decided by the load. On light load or no load, the armature current drawn by the motor is very small.

In case of a d.c. series motor,  $\Phi \propto I_a$  and on no load as  $I_a$ , is small hence flux produced is also very small.

According to speed equation,  $N \propto \frac{1}{\Phi}$  as  $E_b$  is almost constant.

So on very light load or no load as flux is very small, the motor tries to run at dangerously high speed which may damage the motor mechanically. This can be seen from the speed-armature current and the speed-torque characteristics that on low armature current and low torque condition motor shows a tendency to rotate with dangerously high speed.

This is the reason why series motor should never be started on light loads or no load conditions. For this reason it is not selected for belt drives as breaking or slipping of belt causes to throw the entire load off on the motor and made to run motor with no load which is dangerous.



### (c) Characteristics of D.C. Compound Motor

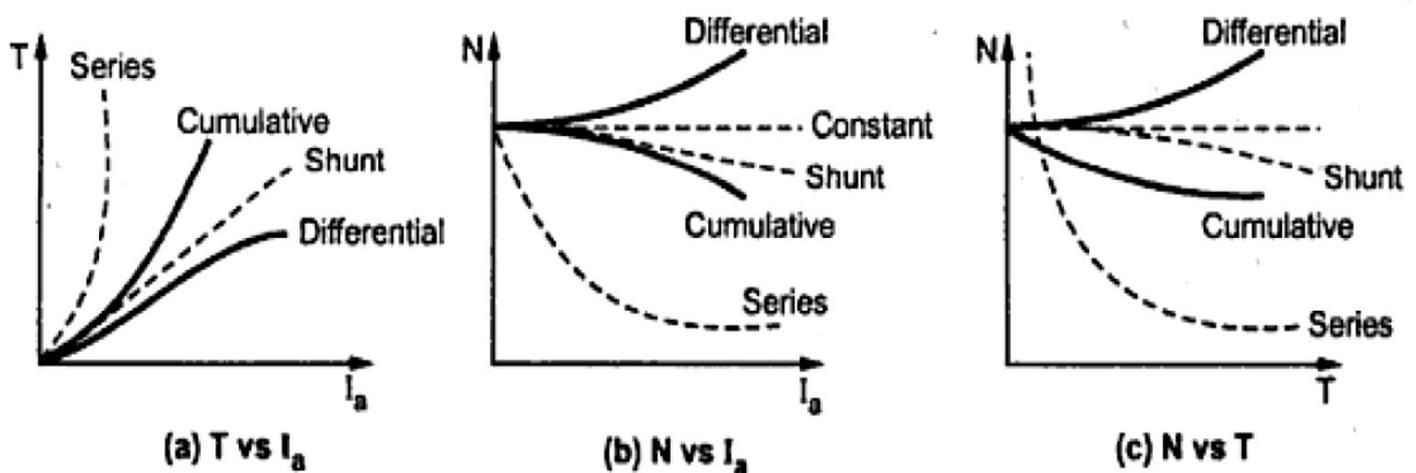
Compound motor characteristics basically depends on the fact whether the motor is cumulatively compound or differential compound. All the characteristics of the compound motor are the combination of the shunt and series characteristic

Cumulative compound motor is capable of developing large amount of torque at low speeds just like series motor. However it is not having a disadvantage of series motor even at light or no load. The shunt field winding produces the definite flux and series flux helps the shunt field flux to increase the total flux level.

So cumulative compound motor can run at a reasonable speed and will not run with dangerously high speed like series motor, on light or no load condition.

In differential compound motor, as two fluxes oppose each other, the resultant flux decreases as load increases, thus the machine runs at a higher speed with increase in the load. This property is dangerous as on full load, the motor may try to run with dangerously high speed. So differential compound motor is generally not used in practice.

The various characteristics of both the types of compound motors cumulative and the differential are shown in the **Figure (24)**(a), (b) and (c).



**Figure (24):** Characteristics of d.c. Compound Motor

The exact shape of these characteristics depends on the relative contribution of series and shunt field windings. If the shunt field winding is more dominant then the characteristics take the shape of the shunt motor characteristics. While if the series field winding is more dominant then the characteristics take the shape of the series characteristics.



**Example 32:** The following results were obtained from a static torque test on a series motor:

|                  |       |       |       |      |
|------------------|-------|-------|-------|------|
| Current (A) :    | 20    | 30    | 40    | 50   |
| Torque (N - m) : | 128.8 | 230.5 | 349.8 | 46.2 |

Deduce the speed/torque curve for the machine when supplied at a constant voltage of 460 V. Resistance of armature and field winding is  $0.5 \Omega$ . Ignore iron and friction losses.

**Solution:**

Taking the case when input current is 20 A, we have

Motor input =  $460 \times 20 = 9,200 \text{ W}$

Field and armature Cu loss =  $20^2 \times 0.5 = 200 \text{ W}$

Ignoring iron and friction losses, output =  $9,200 - 200 = 9,000 \text{ W}$

Now,  $T_{sh} \times 2\pi N = \text{Output in watts.}$

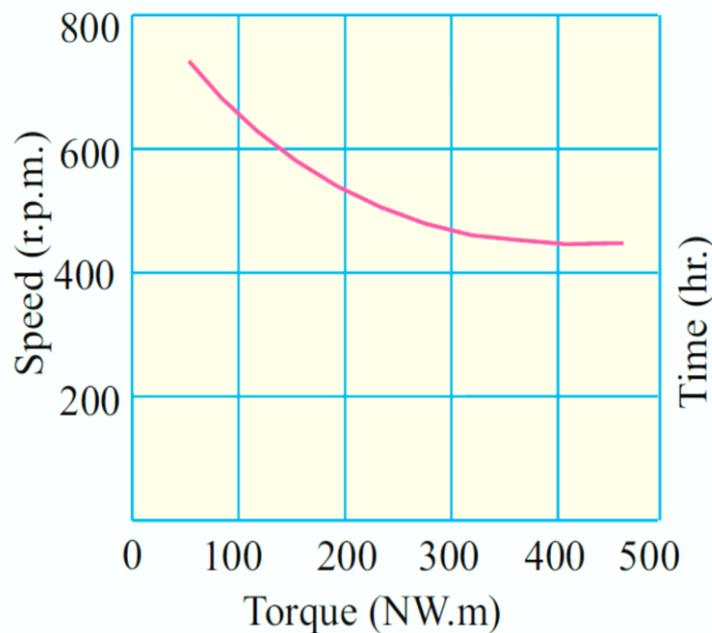
$\therefore 128.8 \times 2\pi \times N = 9,000$

$\therefore N = 9,000 / 2\pi \times 128.8 = 11.12 \text{ r.p.s.} = 667 \text{ r.p.m.}$

Similar calculations for other values of current are tabulated below :

|                 |       |        |        |        |
|-----------------|-------|--------|--------|--------|
| Current (A)     | 20    | 30     | 40     | 50     |
| Input (W)       | 9,200 | 13,800 | 18,400 | 23,000 |
| $I^2R$ loss (W) | 200   | 450    | 800    | 1,250  |
| Output (W)      | 9,200 | 13,350 | 17,600 | 21,850 |
| Speed (r.p.m.)  | 667   | 551    | 480    | 445    |
| Torque (N-m)    | 128.8 | 230.5  | 349.8  | 469.2  |

From these values, the speed/torque curve can be drawn as shown in **Figure (25)**.



**Figure (25)**



## 14. Performance Curves

The four essential characteristics of a shunt motor are shown *i.e.* torque, current speed and efficiency, each plotted as a function of motor output power. These are known as the **performance curves** of a motor.

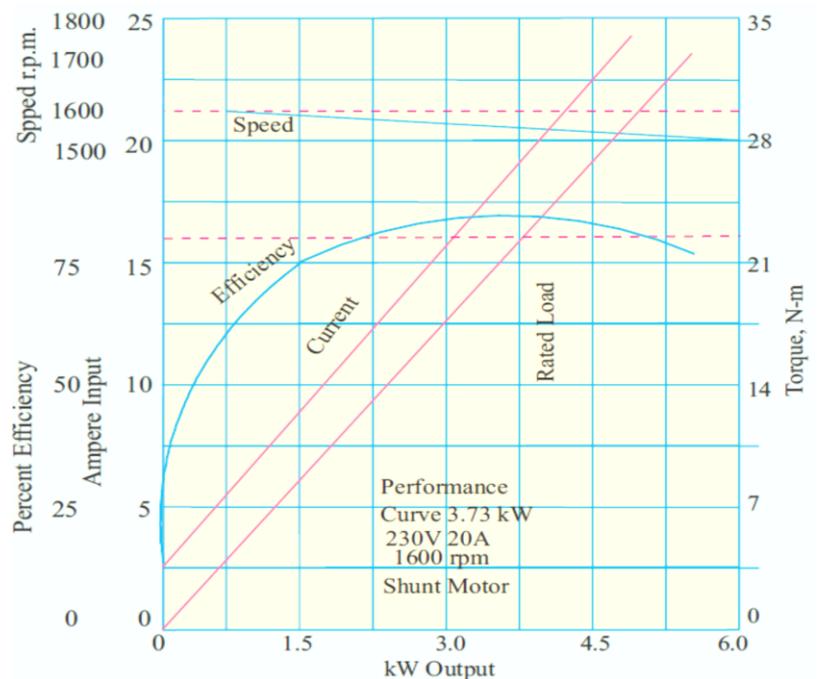
### (a) Shunt Motor

Shunt motor has a definite no-load speed. Hence, it does not 'run away' when load is suddenly thrown off provided the field circuit remains closed. The drop in speed from no-load to full-load is small, hence this motor is usually referred to as **constant speed** motor. The efficiency curve is usually of the same shape for all electric motors and generators. It will be seen from the curves, that a certain value of current is required even when output is zero. The motor input under no-load conditions goes to meet the various losses occurring within the machine.

As compared to other motors, a shunt motor is said to have a lower starting torque. But this should not be taken to mean that a shunt motor is incapable of starting a heavy load. Actually, it means that series and compound motors are capable of starting heavy loads with less excess of current inputs over normal values than the shunt motors and that consequently the depreciation on the motor will be relatively less.

For example, if twice full load torque is required at start, then shunt motor draws twice the full-load current ( $T_a \propto I_a$  or  $I_a \propto T_a$ ) whereas series motor draws only approximately one and a half times the full load current ( $T_a \propto I_a^2$  or  $I_a \propto \sqrt{T_a}$ ).

The shunt motor is widely used with loads that require essentially constant speed but where high starting torques are not needed. Such loads include centrifugal pumps, fans, winding reels conveyors and machine tools etc.



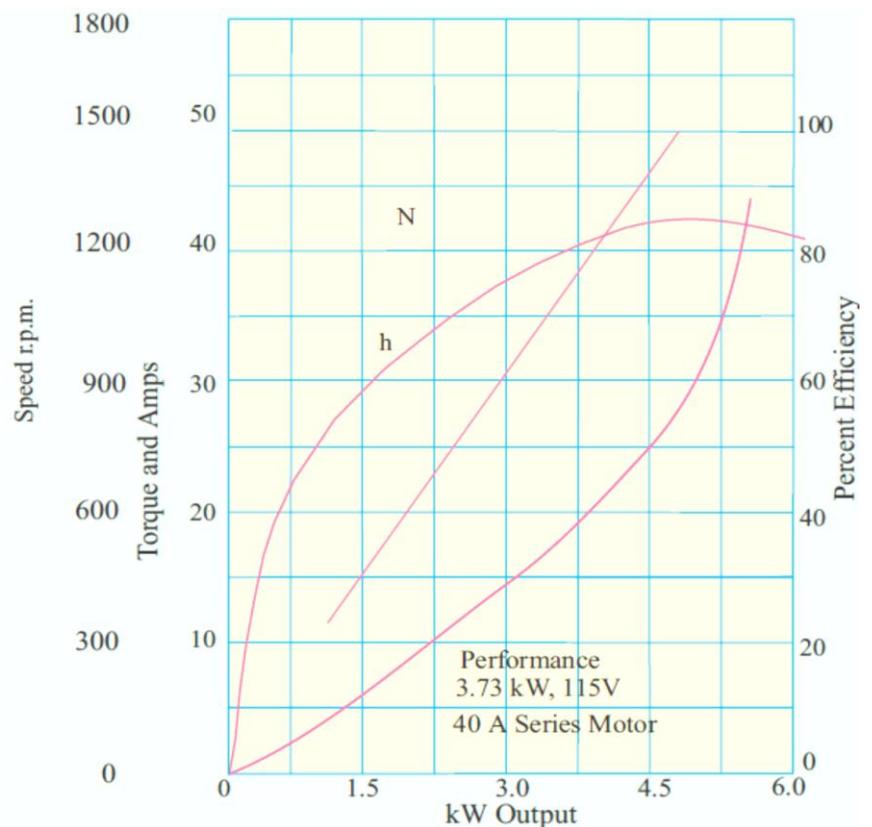
**Figure (26):** Performance curve of shunt motor



### (b) Series Motor

It will be seen that drop in speed with increased load is much more prominent in series motor than in a shunt motor. Hence, a series motor is not suitable for applications requiring a substantially constant speed.

For a given current input, the starting torque developed by a series motor is greater than that developed by a shunt motor. Hence, series motors are used where huge starting torques are necessary *i.e.* for street cars, cranes, hoists and for electric-railway operation. In addition to the huge starting torque, there is another unique characteristic of series motors which makes them especially desirable for traction work *i.e.* when a load comes on a series motor, it responds by decreasing its speed (and hence,  $E_b$ ) and supplies the increased torque with a small increase in current. On the other hand a shunt motor under the same conditions would hold its speed nearly constant and would supply the required increased torque with a large increase of input current. Suppose that instead of a series motor, a shunt motor is used to drive a street car.



**Figure (27):** Performance curve of series motor

### 15. Applications of D.C. Motors



Instead of just stating the applications, the behavior of the various characteristics like speed, starting torque etc, which makes the motor more suitable for the applications, is also stated in the **Table (2)**.

**Table (2): D.C. Motors Characteristics**

| <b>Type of motor</b>    | <b>Characteristics</b>   | <b>Applications</b>   |
|-------------------------|--|---|
| Shunt                   | Approximately constant speed<br>Adjustable speed<br>Medium starting torque (Up to 1.5 F.L. torque) | For driving constant speed line shafting<br>Lathes<br>Centrifugal pumps<br>Machine tools<br>Blowers and fans<br>Reciprocating pumps   |
| Series                  | Variable speed<br>Adjustable varying speed<br>High Starting torque                                 | For traction work i.e. Electric locomotives<br>Rapid transit systems<br>Trolley, cars etc.<br>Cranes and hoists<br>Conveyors  |
| Cumulative Compound     | Variable speed<br>Adjustable varying speed<br>High starting torque                                 | For intermittent high torque loads<br>For shears and punches<br>Elevators<br>Conveyors<br>Heavy planers<br>Heavy planers<br>Rolling mills; Ice machines;<br>Printing presses; Air compressors |
| Differentially Compound | Speed increases as load increases  | Not suitable for any practical application  |

## **16. Losses and Efficiency**

The losses taking place in the motor are the same as in generators. These are



- (i) Copper losses
- (ii) Magnetic losses and
- (iii) Mechanical losses.

The condition for maximum *power* developed by the motor is

$$I_a R_a = \frac{V}{2} = E_b.$$

The condition for maximum *efficiency* is that,

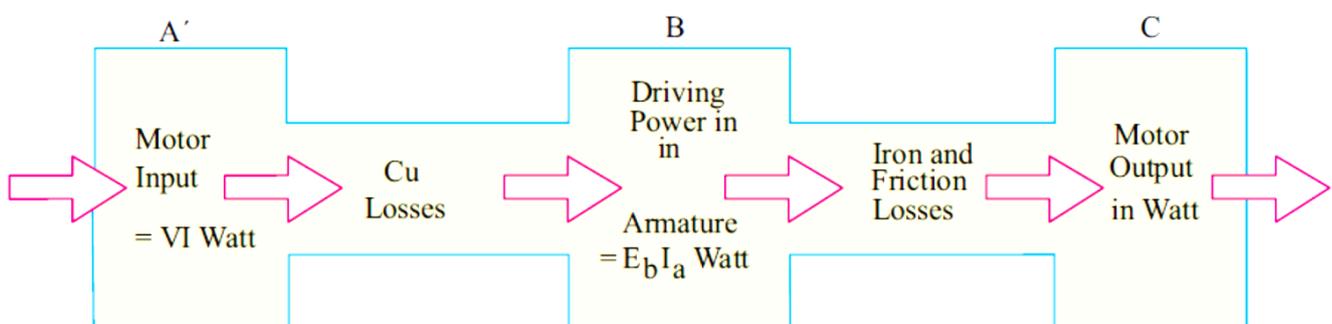
armature Cu losses are equal to constant losses.

### 17. Power Stages

The various stages of energy transformation in a motor and also the various losses occurring in it are shown in the flow diagram of **Figure (28)**.

- Overall or commercial efficiency  $\eta_c = \frac{C}{A}$
- Electrical efficiency  $\eta_e = \frac{B}{A}$
- Mechanical efficiency  $\eta_m = \frac{C}{B}$

It is seen that  $A - B =$  copper losses and  $B - C =$  iron and friction losses.



**Figure (28):** Flow Diagram of Power Stage



Figure (29) shows a comparison between the Flow Diagram of Power Stage for both d.c. generators & d.c. motors.

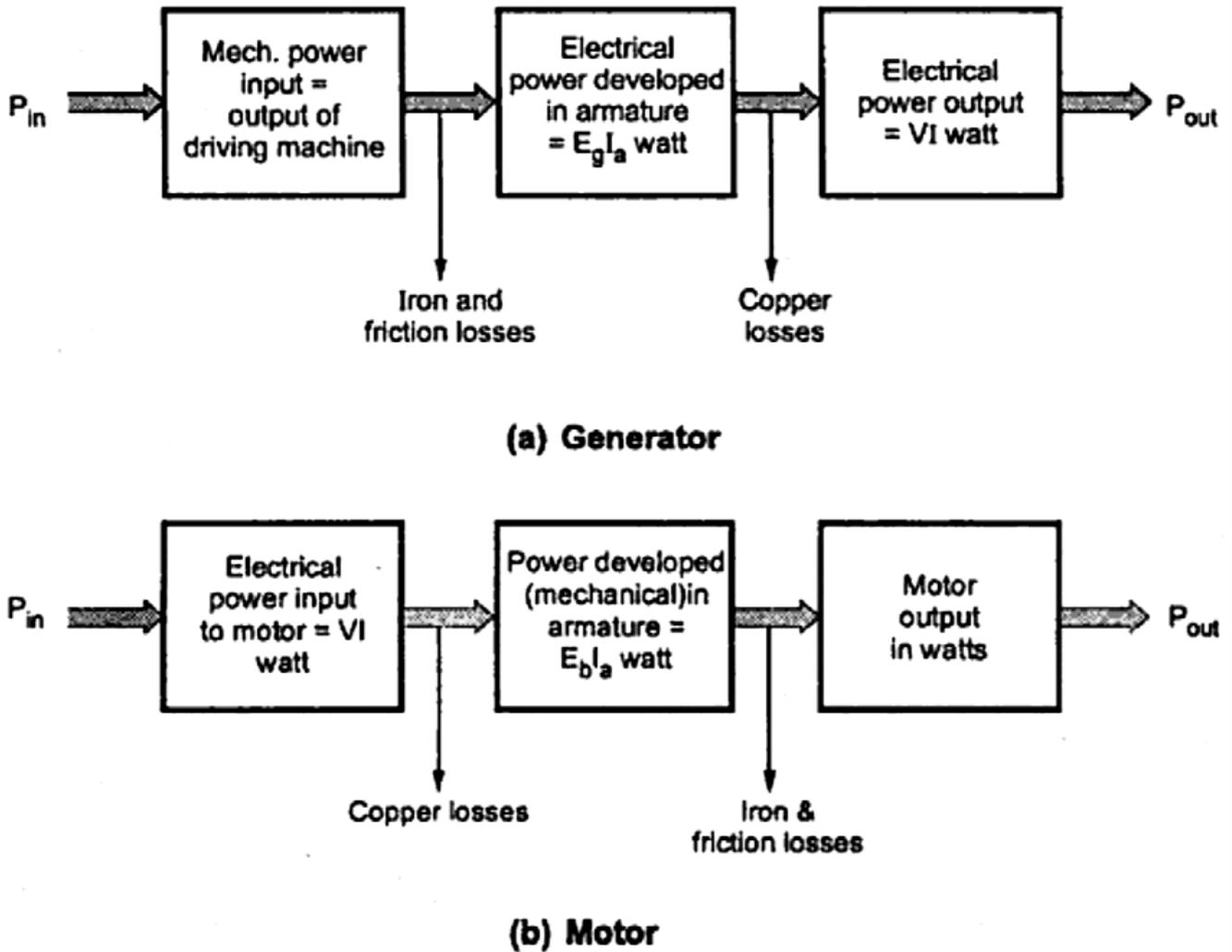


Figure (29): Flow Diagram of Power Stage for generators & motors



**Example 33:** One of the two similar 500-V shunt machines A and B running light takes 3 A. When A is mechanically coupled to B, the input to A is 3.5 A with B unexcited and 4.5 A when B is separately-excited to generate 500 V. Calculate the friction and windage loss and core loss of each machine.

**Solution:**

When machine is running light, this means machine at no load & input power supply losses only, i.e. (i) armature Cu loss (ii) shunt Cu loss (iii) iron loss and (iv) mechanical losses i.e. friction and windage losses.

N.L. losses for each machine equal,  $(P_{\text{no-load input}}) = VI = 500 \times 3 = 1500 \text{ W}$ .

At N.L.,  $(P_{\text{input}} = P_{\text{output}} + W_{\text{stray}} + W_{\text{total cu loss}} = 0 + W_{\text{stray}} + W_{\text{total cu loss}})$

**(a) With B unexcited**

With B unexcited means there is no current flow in windings of machine B hence there is no cu loss, also no flux produce in machine so there is no iron loss. only mechanical loss occur in machine B due to rotation and friction.

Machine A draws current (3.5 A) which,

- (1) 3A supply machine (A) total loss (total cu loss + mech. Loss + iron loss).
- (2)  $(3.5 - 3 = 0.5 \text{ A})$  additional current needed to supply mechanical loss of machine B.

$W_{\text{mech.}} = 0.5 \times 500 = 250 \text{ W}$ , which is the same for machine A & B because these machines are identical & have the same  $W_{\text{mech.}}$ .

or :

$$\text{total loss} = 3.5 \times 500 = 1750 \text{ W}$$

$$W_{\text{mech}} = 1750 - 1500 = 250 \text{ W}$$

**(b) With B excited**

In this case, machine (B) will supply  $W_{\text{mech}}$  &  $W_{\text{iron}}$  and no shunt cu loss occur because it excited separately from external source.

A draws,  $4.5 - 3 = 1.5 \text{ A}$  more current.

Iron and mechanical losses of B ( $W_{\text{mech}} = 1.5 \times 500 = 750 \text{ W}$ )

Iron losses of B ( $W_{\text{iron}} = 750 - 250 = 500 \text{ W}$ )

$W_{\text{cu}} = W_{\text{total}} - W_{\text{stray}} = 1500 - 750 = 750 \text{ W}$



**Example 34:** A 220 V shunt motor has an armature resistance of 0.2 ohm and field resistance of 110 ohm. The motor draws 5 A at 1500 r.p.m. at no load. Calculate the speed and shaft torque if the motor draws 52 A at rated voltage.

**Solution:**

**When working on no-load**

$$I_{sh} = 220/110 = 2 \text{ A}; I_{ao} = 5 - 2 = 3 \text{ A}$$

$$E_{bo} = 220 - 3 \times 0.2 = 219.4 \text{ V}$$

**When working on full-load**

$$I_{a1} = 52 - 2 = 50 \text{ A}$$

$$E_{b1} = 220 - 50 \times 0.2 = 210 \text{ V}$$

$$\frac{E_{b1}}{E_{bo}} = \frac{N_1}{N_o}, \text{ (For shunt motor, } \Phi_1 = \Phi_o)$$

$$\frac{210}{219.4} = \frac{N_1}{1500}$$

$$\therefore N_1 = 1436 \text{ r.p.m.}$$

For finding the shaft torque, we will find the motor output when it draws a current of 52 A. First we will use the no-load data for finding the constant losses of the motor.

$$\text{N.L. motor input} = 220 \times 5 = 1100 \text{ W};$$

$$\text{N.L. arm. Cu loss} = 3^2 \times 0.2 = 1.8 \cong 2 \text{ W}$$

$$\therefore \text{Constant or standing losses of the motor (} W_{\text{stray}} + \text{shunt cu loss)} = 1100 - 2 = 1098 \text{ W}$$

When loaded,

$$\text{F.L. arm. Cu loss} = 50^2 \times 0.2 = 500 \text{ W}$$

$$\text{Hence, total motor losses} = 1098 + 500 = 1598 \text{ W}$$

$$\text{Motor input on load} = 220 \times 52 = 11,440 \text{ W}$$

$$\text{output} = 11,440 - 1598 = 9842 \text{ W}$$

$$\therefore T_{sh} = 9.55 \times (\text{output}/N) = 9.55 \times (9842/1436) = 65.5 \text{ N-m}$$

**Example 35:** 250 V shunt motor on no load runs at 1000 r.p.m. and takes 5 amperes. Armature and shunt field resistances are 0.2 and 250 ohms respectively. Calculate speed when loaded taking a current of 50 A. The armature reaction weakens the field by 3%.

**Solution:**

**When working on no-load**

$$I_{sh} = 250/250 = 1 \text{ A}; I_{ao} = 5 - 1 = 4 \text{ A}, N_o = 1000 \text{ r.p.m.},$$

$$E_{bo} = 250 - 4 \times 0.2 = 249.2 \text{ V}$$

**When working on full-load**

$$I_{a \text{ F.L.}} = 50 - 1 = 49 \text{ A}$$

$$E_{b \text{ F.L.}} = 250 - 49 \times 0.2 = 240.2 \text{ V}$$



$\Phi_{F.L.} = 0.97\Phi_o$  (due to armature reaction).

$$\frac{E_{bF.L.}}{E_{bo}} = \frac{N_{F.L.}}{N_o} \times \frac{\Phi_{F.L.}}{\Phi_o}$$

$$\frac{240.2}{249.2} = \frac{N_{F.L.}}{1000} \times \frac{0.97\Phi_o}{\Phi_o}$$

$N_{F.L.} = 944$  r.p.m.

**Example 36:** A 500 V d.c. shunt motor takes a current of 5 A on no-load. The resistances of the armature and field circuit are 0.22 ohm and 250 ohm respectively. Find (a) the efficiency when loaded and taking a current of 100 A (b) the percentage change of speed.

**Solution:**

**No-Load condition**

$$I_{sh} = 500/250 = 2 \text{ A}; I_{a0} = 5 - 2 = 3 \text{ A}$$

$$E_{bo} = 500 - (3 \times 0.22) = 499.34 \text{ V}$$

$$\text{N.L. Arm. Cu loss} = 3^2 \times 0.22 = 2 \text{ W};$$

$$\text{N.L. motor input} = 500 \times 5 = 2500 \text{ W}$$

$$\text{Constant losses (} W_{\text{stray}} + \text{Shunt cu loss)} = 2500 - 2 = 2498 \text{ W}$$

It is assumed that these losses remain constant under all load conditions.

**Load condition**

(a) Motor current = 100 A;  $I_a = 100 - 2 = 98 \text{ A}$

$$E_b = 500 - (98 \times 0.22) = 478.44 \text{ V}$$

$$\text{Arm. Cu loss} = 98^2 \times 0.22 = 2110 \text{ W},$$

$$\text{Total losses} = 2110 + 2498 = 4608 \text{ W}$$

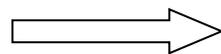
$$\text{Motor input (} P_{\text{input}}) = 500 \times 100 = 50,000 \text{ W},$$

$$\text{Motor output (} P_{\text{output}}) = 50,000 - 4,608 = 45,392 \text{ W}$$

$$\text{Motor } \eta = 45,392/50,000 = 0.908 \text{ or } 90.8\%$$

$$\frac{N_{F.L.}}{N_o} = \frac{E_{bF.L.}}{E_{bo}}$$

$$\therefore \frac{N_{F.L.}}{N_o} = \frac{478.44}{499.34}$$



$$N_{F.L.} = 0.958N_o$$

$$\text{Percentage speed change} = \frac{N_{F.L.} - N_o}{N_o} \times 100 = \frac{0.958N_o - N_o}{N_o} \times 100$$

$$= -4.18\%$$



**Example 37:** A 250 V d.c. shunt motor runs at 1000 r.p.m. while taking a current of 25 A. Calculate the speed when the load current is 50 A if armature reaction weakens the field by 3%. Determine torques in both cases.

$R_a = 0.2 \text{ ohm}$  ;  $R_f = 250 \text{ ohms}$   
Voltage drop per brush is 1 V.

**Solution:**

$$I_{sh} = 250/250 = 1 \text{ A}; I_{a1} = 25 - 1 = 24 \text{ A}$$

$$E_{b1} = 250 - \text{arm. drop} - \text{brush drop} = 250 - 24 \times 0.2 - 2 = 243.2 \text{ V}$$

$$I_{a2} = 50 - 1 = 49 \text{ A}$$

$$E_{b2} = 250 - 49 \times 0.2 - 2 = 238.2 \text{ V}$$

$$\Phi_2 = 0.97\Phi_1$$

$$\frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1} \times \frac{\Phi_2}{\Phi_1}$$

$$\frac{238.2}{243.2} = \frac{N_2}{1000} \times \frac{0.97\Phi_1}{\Phi_1}$$

$$N_2 = \mathbf{1010 \text{ r.p.m.}}$$

$$T_{a1} = 9.55 (E_{b1} I_{a1}/N_1) = 9.55 \times 243.2 \times 24/1000 = \mathbf{55.7 \text{ N-m}}$$

$$T_{a2} = 9.55 (E_{b2} I_{a2}/N_2) = 9.55 \times (238.2 \times 49/1010) = \mathbf{110.4 \text{ r.p.m.}}$$

**Example 38:** A d.c. shunt machine while running as generator develops a voltage of 250 V at 1000 r.p.m. on no-load. It has armature resistance of 0.5  $\Omega$  and field resistance of 250  $\Omega$ . When the machine runs as motor, input to it at no-load is 4 A at 250 V. Calculate the speed and efficiency of the machine when it runs as a motor taking 40 A at 250 V. Armature reaction weakens the field by 4 %.

**Solution:**

**When running as generator on N.L.:**

$$N_{go} = 1000 \text{ r.p.m.}, E_{go} = 250 \text{ V (generated voltage for generator at N.L. = } E_g)$$

**When running as motor on N.L.:**

$$I_{sh} = 250/250 = 1 \text{ A}, I_{a0} = I_{Lo} - I_{sh} = 4 - 1 = 3 \text{ A}$$

$$E_{bo} = V - I_{a0}R_a = 250 - (3 \times 0.5) = 248.5 \text{ V}$$

$$\frac{E_{bo}}{E_g} = \frac{N_{mo}}{N_{go}} \times \frac{\Phi_{mo}}{\Phi_{go}}, (\Phi_{go} = \Phi_{mo})$$

$$\frac{248.5}{250} = \frac{N_{mo}}{1000}$$

$$\therefore N_{mo} = \mathbf{994 \text{ r.p.m.}}$$



### When running as motor on F.L.:

$$I_a = I_L - I_{sh} = 40 - 1 = 39 \text{ A}$$

$$E_b = V - I_a R_a = 250 - (39 \times 0.5) = 230.5 \text{ V}$$

$$\Phi_m = 0.96 \Phi_{m0}$$

$$\frac{E_b}{E_{b0}} = \frac{N_m}{N_{b0}} \times \frac{\Phi_m}{\Phi_{b0}}$$

$$\frac{230.5}{248.5} = \frac{N_m}{994} \times \frac{0.96 \Phi_{m0}}{\Phi_{m0}}$$

$$\therefore N_m = \mathbf{960.4 \text{ r.p.m.}}$$

Or:

### When running as generator on N.L.:

$$N_{go} = 1000 \text{ r.p.m.}, \quad E_{go} = 250 \text{ V (generated voltage for generator at N.L. = } E_g)$$

### When running as motor on F.L.:

$$I_a = I_L - I_{sh} = 40 - 1 = 39 \text{ A}$$

$$E_b = V - I_a R_a = 250 - (39 \times 0.5) = 230.5 \text{ V}$$

$$\Phi_m = 0.96 \Phi_{go}, \text{ ( because } (\Phi_{go} = \Phi_{m0})$$

$$\frac{E_b}{E_{go}} = \frac{N_m}{N_{go}} \times \frac{\Phi_m}{\Phi_{go}}$$

$$\frac{230.5}{250} = \frac{N_m}{1000} \times \frac{0.96 \Phi_{go}}{\Phi_{go}}$$

$$\therefore N_m = \mathbf{960.4 \text{ r.p.m.}}$$

### Efficiency

No-load input represents motor losses which consists of

(a) armature Cu loss =  $I_a^2 R_a$  which is variable.

(b) constant losses  $W_c$  which consists of

(i) shunt Cu loss

(ii) magnetic losses and

(iii) mechanical losses.

$$\text{N.L. input or total losses} = 250 \times 4 = 1000 \text{ W}$$

$$\text{N.L. Arm. Cu loss} = I_a^2 R_a = 3^2 \times 0.5 = 4.5 \text{ W,}$$

$$\therefore W_c \text{ ( } W_{\text{stray}} + \text{ Shunt cu loss)} = 1000 - 4.5 = 995.5 \text{ W}$$

When motor draws a line current of 40 A, its armature current is  $(40 - 1) = 39 \text{ A}$

F.L. Arm. Cu loss =  $39^2 \times 0.5 = 760.5 \text{ W}$ ; Total losses =  $760.5 + 995.5 = 1756 \text{ W}$

Input =  $250 \times 40 = 10,000 \text{ W}$ ; output =  $10,000 - 1756 = 8,244 \text{ W}$

$$\therefore \eta = 8,244 \times 100 / 10,000 = \mathbf{82.44\%}$$



**Example 39:** The armature winding of a 4-pole, 250 V d.c. shunt motor is lap connected. There are 120 slots, each slot containing 8 conductors. The flux per pole is 20 mWb and current taken by the motor is 25 A. The resistance of armature and field circuit are 0.1 and 125  $\Omega$  respectively. If the rotational losses amount to be 810 W find,  
(i) gross torque (ii) useful torque and (iii) efficiency.

**Solution:**

$$I_{sh} = 250/125 = 2A; I_a = 25 - 2 = 23 A; E_b = 250 - 23 \times 0.1 = 247.7 V, Z_{total} = 120 \times 8 = 960$$

$$\text{Now, } E_b = \frac{\Phi Z N}{60} \left( \frac{P}{A} \right) \quad \longrightarrow \quad 247.7 = \frac{20 \times 10^{-3} \times 960 \times N}{60} \left( \frac{4}{4} \right)$$

$$\therefore N = \mathbf{774 \text{ r.p.m.}}$$

$$\text{(i) Gross torque or armature torque } T_a = 9.55 \times \frac{E_b I_a}{N} = 9.55 \times \frac{247.7 \times 23}{774} = \mathbf{70.3 \text{ N} \cdot \text{m}}$$

(ii) Arm Cu loss =  $23^2 \times 0.1 = 53 \text{ W}$ , Shunt Cu loss =  $250 \times 2 = 500 \text{ W}$   
Rotational losses = 810 W, Total motor losses =  $810 + 500 + 53 = 1363 \text{ W}$   
Motor input =  $250 \times 25 = 6250 \text{ W}$ , Motor output =  $6250 - 1363 = 4887 \text{ W}$

$$T_{sh} = 9.55 \times \frac{P_{output}}{N} = 9.55 \times \left( \frac{4887}{774} \right) = \mathbf{60.3 \text{ N} \cdot \text{m}}$$

$$\text{(iii) Efficiency} = 4887/6250 = 0.782 = \mathbf{78.2\%}$$

**Example 40:** A 20-hp (14.92 kW); 230-V, 1150-r.p.m. 4-pole, d.c. shunt motor has a total of 620 conductors arranged in two parallel paths and yielding an armature circuit resistance of 0.2  $\Omega$ . When it delivers rated power at rated speed, it draws a line current of 74.8 A and a field current of 3 A. Calculate (i) the flux per pole (ii) the torque developed (iii) the rotational losses (iv) total losses expressed as a percentage of power.

**Solution:**

$$I_a = 74.8 - 3 = 71.8 A; E_b = 230 - 71.8 \times 0.2 = 215.64 V$$

$$\text{(i) Now, } E_b = \frac{\Phi Z N}{60} \left( \frac{P}{A} \right) \quad \longrightarrow \quad 215.64 = \frac{\Phi \times 620 \times 1150}{60} \left( \frac{4}{2} \right)$$

$$\therefore \Phi = \mathbf{9 \text{ m.Wb.}}$$

$$\text{(ii) Armature Torque, } T_a = 9.55 \times \frac{E_b I_a}{N} = 9.55 \times \frac{215.64 \times 71.8}{1150} = \mathbf{128.8 \text{ N} \cdot \text{m}}$$

(iii) Driving power in armature =  $E_b I_a = 215.64 \times 71.8 = 15,483 \text{ W}$ ,  
20-hp (14.92 kW) represent ( $P_{output}$ ).  
Rotational losses =  $15,483 - 14,920 = \mathbf{563 \text{ W}}$

(iv) Motor input =  $VI = 230 \times 74.8 = 17,204 \text{ W}$ , Total loss =  $17,204 - 14,920 = 2,284 \text{ W}$   
Losses expressed as percentage of power input =  $2284/17,204 = 0.133$  or  $\mathbf{13.3\%}$



**Example 41:** A 7.46 kW, 250-V shunt motor takes a line current of 5 A when running light. Calculate the efficiency as a motor when delivering full load output, if the armature and field resistance are 0.5 Ω and 250 Ω respectively. At what output power will the efficiency be maximum ? Is it possible to obtain this output from the machine ?

**Solution:**

**When running as motor on light load (N.L.):**

$$I_{sh} = 250/250 = 1 \text{ A}, I_{ao} = I_{Lo} - I_{sh} = 5 - 1 = 4 \text{ A}$$

$$E_{bo} = V - I_{ao}R_a = 250 - (4 \times 0.5) = 248 \text{ V}$$

$$P_{output} + W_{stray} = P_{mo} \quad \longrightarrow \quad 0 + W_{stray} = P_{mo}$$

$$\therefore W_{stray} = P_{mo} = E_{bo}I_{ao} = 248 \times 4 = \mathbf{992 \text{ W}}$$

**When running as motor on F.L.:**

$$P_{output} = 7.46 \text{ kW}, P_m = P_{output} + W_{stray} = 7460 + 992 = \mathbf{8452 \text{ W}}$$

$$P_m = E_b I_a = (V - I_a R_a) I_a = (250 - 0.5 I_a) I_a$$

$$\therefore P_m = (250 - 0.5 I_a) I_a$$

$$8452 = 250 I_a - 0.5 I_a^2$$

$$\therefore 0.5 I_a^2 - 250 I_a + 8452 = 0$$

$$\therefore I_a = \frac{250 \pm \sqrt{(-250)^2 - 4 \times 0.5 \times 8452}}{2 \times 0.5}$$

Either :  $I_a = 463.5 \text{ A}$  (this value is neglected because it is very large)

Or:  $I_a = \mathbf{36.5 \text{ A}}$  (True)

$$\therefore \text{F.L. input current} = I_a + I_{sh} = 36.5 + 1 = 37.5 \text{ A}$$

$$P_{input} = V I_L = 250 \times 37.5 = \mathbf{9375 \text{ W}}$$

$$P_{output} = \mathbf{7460 \text{ W}}$$

$$\therefore \text{F.L. efficiency} = (7460 / 9375) \times 100 = \mathbf{79.6\%}$$

Now, efficiency is maximum when armature Cu loss equals constant loss.

$$W_{constant} = W_{stray} + \text{shunt cu loss} = 992 + V I_{sh} = 992 + 250 \times 1 = \mathbf{1242 \text{ W}}$$

$$\text{F.L. Arm. Cu loss} = I_a^2 R_a = I_a^2 \times 0.5$$

$$\therefore \text{For max. efficiency, } I_a^2 \times 0.5 = 1242 \quad \longrightarrow \quad I_a = 49.84 \text{ A}$$

$$\therefore \text{Armature input} = 250 \times 49.84 = 12,460 \text{ W}$$

$$\text{Armature Cu loss} = 49.84^2 \times 0.5 = 1242 \text{ W}; \text{ Iron and friction losses} = 992 \text{ W}$$

$$\therefore \text{Armature output} = 12,460 - (1,242 + 992) = 10,226 \text{ W}$$

$$\therefore \text{Output power} = 10,226 \text{ W} = \mathbf{10.226 \text{ kW}}$$

As the input current for maximum efficiency is beyond the full-load motor current, it is never realized in practice.



**Example 42:** d.c. series motor drives a load, the torque of which varies as the square of the speed. Assuming the magnetic circuit to be remain unsaturated and the motor resistance to be negligible, estimate the percentage reduction in the motor terminal voltage which will reduce the motor speed to half the value it has on full voltage. What is then the percentage fall in the motor current and efficiency ? Stray losses of the motor may be ignored.

**Solution:**

In general,  $T_a \propto \Phi I_a \propto I_a^2$  (for series motor because  $\Phi \propto I_a$ )

. Also,  $T_a \propto N^2$ .

Hence  $N^2 \propto I_a^2$  or  $N \propto I_a$

$$\therefore N_1 \propto I_{a1}$$

And  $N_2 \propto I_{a2}$

$$\therefore \frac{N_2}{N_1} = \frac{I_{a2}}{I_{a1}}$$

Since,  $\frac{N_2}{N_1} = \frac{1}{2}$

$$\therefore \frac{I_{a2}}{I_{a1}} = \frac{1}{2} \quad \text{or} \quad I_{a2} = \frac{I_{a1}}{2}$$

Let  $V_1$  and  $V_2$  be the voltages across the motor in the two cases. Since motor resistance is negligible,

$E_{b1} = V_1$  and  $E_{b2} = V_2$ .

Also  $\Phi_1 \propto I_{a1}$  and  $\Phi_2 \propto I_{a2}$  or  $\frac{\Phi_1}{\Phi_2} = \frac{I_{a1}}{I_{a2}} = I_{a1} \times \frac{2}{I_{a1}} = 2$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

$$\frac{1}{2} = \frac{V_2}{V_1} \times 2$$

$$\frac{V_2}{V_1} = \frac{1}{4}$$

$$\therefore \text{Percentage reduction in voltage} = \frac{V_1 - V_2}{V_1} \times 100 = \frac{4-1}{4} \times 100 = 0.75 \times 100 = \mathbf{75\%}$$

$$\text{Percentage change in motor current} = \frac{I_{a1} - I_{a2}}{I_{a1}} \times 100 = \frac{I_{a1} - \frac{I_{a1}}{2}}{I_{a1}} \times 100 = \mathbf{50\%}$$



**Example 43:** A 6-pole, 500-V wave-connected shunt motor has 1200 armature conductors and useful flux/pole of 20 mWb. The armature and field resistance are 0.5  $\Omega$  and 250  $\Omega$  respectively. What will be the speed and torque developed by the motor when it draws 20 A from the supply mains ? Neglect armature reaction. If magnetic and mechanical losses amount to 900 W, find (i) useful torque (ii) output in kW and (iii) efficiency at this load.

**Solution:**

$$I_{sh} = 500/250 = 2 \text{ A} \therefore I_a = 20 - 2 = 18 \text{ A}$$

$$\therefore E_b = 500 - (18 \times 0.5) = 491 \text{ V}$$

$$\text{Now, } E_b = \frac{\Phi Z N}{60} \left( \frac{P}{A} \right) \quad \Longrightarrow \quad 491 = \frac{20 \times 10^{-3} \times 1200 \times N}{60} \left( \frac{6}{2} \right)$$

$$\therefore N = \mathbf{410 \text{ r.p.m. (approx.)}}$$

$$\text{Now, } T_a = 9.55 \times \frac{E_b I_a}{N} = 9.55 \times \frac{491 \times 18}{410} = \mathbf{206 \text{ N-m}}$$

$$\text{Armature Cu loss} = 18^2 \times 0.5 = 162 \text{ W}; \text{ Field Cu loss} = 500 \times 2 = 1000 \text{ W}$$

$$\text{Iron and friction loss} = 900 \text{ W}; \text{ Total loss} = 162 + 1000 + 900 = 2,062 \text{ W}$$

$$\text{Motor input} = 500 \times 20 = 10,000 \text{ W}$$

$$\text{(i) } T_{sh} = 9.55 \times \frac{P_{\text{output}}}{N} = 9.55 \times \left( \frac{7938}{410} \right) = \mathbf{184.8 \text{ N-m}}$$

$$\text{(ii) Output} = 10,000 - 2062 = \mathbf{7,938 \text{ kW}}$$

$$\text{(iii) } \% \eta = \frac{P_{\text{output}}}{P_{\text{input}}} \times 100 = \frac{7938}{10000} \times 100 = \mathbf{79.4\%}$$

**Example 44:** A 50-h.p. (37.3 kW), 460-V d.c. shunt motor running light takes a current of 4 A and runs at a speed of 660 r.p.m. The resistance of the armature circuit (including brushes) is 0.3  $\Omega$  and that of the shunt field circuit 270  $\Omega$ . Determine when the motor is running at full load (i) the current input (ii) the speed. Determine the armature current at which efficiency is maximum. Ignore the effect of armature reaction.

**Solution:**

**When running as motor on light load (N.L.):**

$$I_{sh} = 460/270 = 1.7 \text{ A}, I_{a0} = I_{L0} - I_{sh} = 4 - 1.7 = 2.3 \text{ A}$$

$$E_{b0} = V - I_{a0} R_a = 460 - (2.3 \times 0.3) = 459.3 \text{ V} \cong 460 \text{ V}$$

$$P_{\text{output}} + W_{\text{stray}} = P_{\text{mo}} \quad \Longrightarrow \quad 0 + W_{\text{stray}} = P_{\text{mo}}$$

$$\therefore W_{\text{stray}} = P_{\text{mo}} = E_{b0} I_{a0} = 460 \times 2.3 = \mathbf{1058 \text{ W}}$$



**When running as motor on F.L.:**

$$P_{\text{output}} = 37.3 \text{ kW}, \quad P_m = P_{\text{output}} + W_{\text{stray}} = 37300 + 1058 = \mathbf{38358 \text{ W}}$$

$$P_m = E_b I_a = (V - I_a R_a) I_a = (460 - 0.3 I_a) I_a$$

$$\therefore P_m = (460 - 0.3 I_a) I_a$$

$$38358 = 460 I_a - 0.3 I_a^2$$

$$\therefore 0.3 I_a^2 - 460 I_a + 38358 = 0$$

$$\therefore I_a = \frac{460 \pm \sqrt{(-460)^2 - 4 \times 0.3 \times 38358}}{2 \times 0.3}$$

Either :  $I_a = 1444.84 \text{ A}$  (this value is neglected because it is very large)

Or:  $I_a = \mathbf{88.5 \text{ A}}$  (True)

(i)  $\therefore$  F.L. input current =  $I_a + I_{\text{sh}} = 88.5 + 1.7 = \mathbf{90.2 \text{ A}}$

(ii)  $E_b = 460 - (88.5 \times 0.3) = 433.5 \text{ V}$

$$\frac{N_2}{N_o} = \frac{E_b}{E_{bo}} \quad \longrightarrow \quad \frac{N_2}{660} = \frac{433.5}{459.3}$$

$$\therefore N_2 = 660 \times (433.5/459.3) = \mathbf{623 \text{ r.p.m.}}$$

Now, efficiency is maximum when armature Cu loss equals constant loss.

$$W_{\text{constant}} = W_{\text{stray}} + \text{shunt cu loss} = 1058 + V I_{\text{sh}} = 1058 + 460 \times 1.7 = \mathbf{1840 \text{ W}}$$

$$\text{F.L. Arm. Cu loss} = I_a^2 R_a = I_a^2 \times 0.3$$

$$\therefore \text{For max. efficiency, } I_a^2 \times 0.3 = 1840 \quad \longrightarrow \quad I_a = 78.32 \text{ A}$$

### Tutorial Problems (3)

[1] A 4-pole 250-V, d.c. series motor has a wave-wound armature with 496 conductors. Calculate

(a) the gross torque

(b) the speed

(c) the output torque and

(d) the efficiency, if the motor current is 50 A

The value of flux per pole under these conditions is 22 mWb and the corresponding iron, friction and windage losses total 810 W. Armature resistance = 0.19  $\Omega$ , field resistance = 0.14  $\Omega$ . [(a) **173.5 N-m** (b) **642 r.p.m.** (c) **161.4 N-m** (d) **86.9%**]



[2] On no-load, a shunt motor takes 5 A at 250 V, the resistances of the field and armature circuits are  $250 \Omega$  and  $0.1 \Omega$  respectively. Calculate the output power and efficiency of the motor when the total supply current is 81 A at the same supply voltage. **[18.5 kW; 91%]**

[3] A 230 V series motor is taking 50 A. Resistance of armature and series field windings is  $0.2 \Omega$  and  $0.1 \Omega$  respectively. Calculate :

- (a) brush voltage (b) back e.m.f.  
(c) power wasted in armature (d) mechanical power developed  
[(a) **10 V** (b) **205 V** (c) **500 W** (d) **13.74 h.p. (10.25 kW)**]

[4] Calculate the shaft power of a series motor having the following data; overall efficiency 83.5%, speed 550 r.p.m. when taking 65 A; motor resistance  $0.2 \Omega$ , flux per pole 25 mWb, armature winding lap with 1200 conductor. **(15.66 kW)**

[5] A shunt motor running on no-load takes 5 A at 200 V. The resistance of the field circuit is  $150 \Omega$  and of the armature  $0.1 \Omega$ . Determine the output and efficiency of motor when the input current is 120 A at 200 V. State any conditions assumed. **(89.8%)**

[6] A d.c. shunt motor with interpoles has the following particulars :

Output power ; 8,952 kW, 440-V, armature resistance  $1.1 \Omega$ , brush contact drop 2 V, interpole winding resistance  $0.4 \Omega$  shunt resistance  $650 \Omega$ , resistance in the shunt regulator  $50 \Omega$ . Iron and friction losses on full-load 450 W. Calculate the efficiency when taking the full rated current of 24 A. **(85%)**

[7] A d.c. series motor on full-load takes 50 A from 230 V d.c. mains. The total resistance of the motor is  $0.22 \Omega$ . If the iron and friction losses together amount to 5% of the input, calculate the power delivered by the motor shaft. Total voltage drop due to the brush contact is 2 V. **(10.275 kW)**

[8] A 2-pole d.c shunt motor operating from a 200 V supply takes a full-load current of 35 A, the no load current being 2 A. The field resistance is  $500 \Omega$  and the armature has a resistance of  $0.6 \Omega$ . Calculate the efficiency of the motor on full-load. Take the brush drop as being equal to 1.5 V per brush arm. Neglect temperature rise. **(82.63%)**

